Expansion of Autonomous Microgrids in Active Distribution Networks

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***Abstract*—This paper presents an approach to transform the active distribution network with distributed energy resources (DERs) into multiple autonomous microgrids. The distribution network consists of several generation resources and demand entities, that are clustered into autonomous microgrids. The proposed problem is formulated as a bi-level optimization problem that leverages the Eigen decomposition in the graph spectra of the distribution network to determine the boundaries for microgrids and a mixed-integer programming problem that minimizes the expansion cost within microgrids. The presented approach is evaluated in a case study for a distribution network considering the imposed reliability constraints. The outcomes indicate the effectiveness of the proposed algorithm to determine the expansion strategies to form autonomous microgrids in active distribution networks.**

***Index Terms*—microgrids, distribution network, reliability, graph theory.**

NOMENCLATURE

Indices and variables:

b, b'	Bus index
$B_{b,b'}^{t,s}$	Element of susceptance matrix connecting buses b and b' at time t in scenario s
g	Index of grid connection points
$G_{b,b'}^{t,s}$	Element of conductance matrix connecting buses b *and* b' at time t in scenario s
i	Index of distributed generation unit
$P_{b,t}^{d,s}, Q_{b,t}^{d.s}$	Real/reactive demand served at bus b at time t in scenario s
$P_i^{t,s}, Q_i^{t,s}$	Real/reactive generation dispatch of unit i at time t in scenario s
$P_g^{t,s}, Q_g^{t,s}$	Real/reactive power dispatch of main grid g at time t in scenario s
$P_{b,t}^{inj,s}, Q_{b,t}^{inj,s}$	Real/reactive power injection at bus b at time t in scenario s
$PL_{b,b'}^{t,s}, QL_{b,b'}^{t,s}$	Real/reactive power flow between buses b and b' at time t in scenario s
$SL_{b,b'}^{t,s}$	Apparent power flow between buses b and b' at time t in scenario s
s	Scenario index
t	Index for time period in the load duration curve
$u_{b,b'}$	Decision variable for the connectivity between buses b and b'
$UX_i^{t,s}$	Generator's outage status at time t in scenario s, 1 if available, otherwise 0
$UY_{b,b'}^{t,s}$	Distribution line's outage status at time t in scenario s, 1 if available, otherwise 0
$UZ_g^{t,s}$	The main grid connection's outage status at time t in scenario s, 1 if available, otherwise 0
$V_b^{t,s}$	Voltage magnitude at bus b at time t in scenario s
$\theta_b^{t,s}$	Voltage angle at bus b at time t in scenario s

Constants:

$c_g^{t,s}$	Price of electricity at time t in scenario s
$EENS_b$	Expected energy not supplied at bus b
$F_{c,i}$	Production cost function for thermal unit i
GG_b	Set of units that are connected to bus b
GR_b	Set of the utility grid connection to bus b
$LOEP_b$	Loss of energy probability at bus b
NB	Total number of buses
NG	Total number of units
NL	Total number of lines
NS	Total number of scenarios
NT	Total number of time periods under study
$P_i^{\max}, Q_i^{\max}$	Real/reactive generation capacity of unit i
$P_{b,t}^{D,s}, Q_{b,t}^{D,s}$	Real/reactive power demand at bus b at time t in scenario s
$SL_{b,b'}^{\max}$	Maximum capacity of line connecting buses b and b'
$VOLL_b$	Value of lost load at bus b
V^{max}, V^{min}	Maximum and minimum voltage magnitude
$\theta^{\max}, \theta^{\min}$	Maximum and minimum voltage angle
$x_{b,b'}$	Reactance of the line between buses b and b'
$r_{b,b'}$	Resistance of the line between buses b and b'

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$y_{b,b'}$	Admittance of the line between buses b and b'
ρ^s	Probability of scenario s
σ_t	Duration of time period in load duration curve
$\xi_{b.b'}$	Auxiliary parameter

I. Introduction

RELIABILITY of energy supply in distribution networks is dependent on the availability of the distribution feeders, as they have radial topology [1]. A fault or failure in the line or feeder within the radial network will lead to demand curtailment. These failures are caused randomly by component outages in the network [2]. Self-healing approaches including fault isolation, network reconfiguration by remote-controlled tie-switches, and load restoration procedures are employed to minimize the energy curtailment in the distribution networks [3], [4]. Investment in such capabilities would increase the overall reliability and resilience of the network and differentiate the quality of service for the customers in the distribution networks [5].

The increase in the installed capacity of DERs requires distributed and efficient control of such resources, which is facilitated by forming microgrids [6]. Microgrids are composed of DERs and demands with distinct boundaries that are connected to the utility grid through the point of common coupling (PCC) or operated in island mode [7]. Forming microgrids can help to improve the restoration capability of the distribution networks [8], [9]. Islanding in microgrids reduces the adverse effects of outages and power quality deterioration on local demands. Moreover, developing efficient control schemes in microgrids facilitates the trade of electricity and ancillary services in distribution market [10].

While increasing the number of microgrids promotes the service reliability, facilitates distributed control over the generation and demand resources and reduces the investment cost to improve the reliability and resilience; defining the boundaries for microgrids in distribution networks to serve the customers is a challenging task. In [11]-[12], a number of methodologies are proposed to address the allocation and sizing of DERs in the distribution networks. DERs increase the reliability of energy supply and transforming the distribution network into several self-sufficient microgrids by defining PCCs would help to designate the energy resources to the load points in contingencies.

In this context, partitioning the electricity network into several sub-networks using graphical and Eigenvalue sensitivity-based approaches is discussed in [13]. The graph spectra are assessed using a sensitivity-based approach in order to partition the network and analyze the dynamic properties of the power system in the procured sub-networks. However, the base for network partitioning does not capture the power flow constraints and the reliability of the power system. Several heuristic approaches were proposed to sectionalize the distribution network in order to form autonomous microgrids for various applications [14]-[16]. These heuristic approaches are utilized to divide an existing distribution network into several autonomous microgrids to achieve several objectives including reliability enhancement, supply security, and avoiding cascading failures. A heuristic and conservative intentional islanding procedure is proposed in [14], which requires that the generation capacity of each microgrid be at least 30% more than its demand. In [15], network design scheme to establish microgrids is presented to enhance the reliability and energy supply security. An islanding-based approach for the self-healing system is proposed in [16] to avoid cascading failure events as a result of the component failures in the network. Here, similar to [17], the heuristic coarsening and un-coarsening processes are used to cluster the vertices and partition the distribution network graph. In [18], an approach to form autonomous microgrids in the distribution network is proposed to restore critical demands in real-time operation. The objective is to maximize the restored critical demands, while the microgrids are energized by DERs. Partitioning the distribution network into microgrids in order to facilitate self-healing during severe disturbances is addressed in [19]. A stochastic rolling horizon framework is employed to sectionalize the on-outage distribution network into multiple microgrids, while the supply adequacy constraints are satisfied. In [20], an approach is employed to cluster the distribution network into multiple microgrids to ensure the supply adequacy in the network. The faulted distribution network is partitioned in order to minimize the energy imbalances in each microgrid and improve the control robustness. Greedy and Ant colony algorithms are employed in [21]-[22] to partition the distribution network into several self-adequate microgrids.

II. Motivations and Objectives

While earlier research is focused on maintaining the supply adequacy in contingencies, this paper addresses the supply adequacy capturing the reliability requirements of the demands and the quality of service at demand buses is ensured by forming microgrids and expansion of the distribution network. As a consequence, the developed microgrids provide heterogeneous energy supply reliability at demand buses considering the outages in generation and distribution assets.

Sectionalizing the distribution network to provide boundaries for the DERs to serve the consumers will increase the reliability of energy supply in contingencies as DERs are dispatched as backup generation resources in the distribution network. Moreover, sectionalizing the distribution network to form microgrids will improve the robustness in the control of generation and demand assets. In this context, DERs operate within the microgrids to ensure the reliability of energy supply. A fundamental challenge in controlling DERs is the significant number of control variables in the distribution network [23]. When DERs serve the customers in contingencies, a failure in a complex control system can shut down the entire system considering the fact that the generation resources and their inertia are limited in the distribution network. In order to mitigate the disturbance propagation and reduce the vulnerability of the distribution networks to voltage sags, swells, faults, and the uncertainties in demand and supply balance, microgrids are formed and islanded in emergency conditions. Islanding improves the quality of energy supply during disturbances in the network. Minimizing the number of generation, distribution and demand assets in each microgrid – to provide more robust control – will maximize the number of microgrids formed in the network. However, the reliability requirement of the demand will limit the size of the microgrids. Larger microgrids with more DERs provide higher service reliability. Moreover, expanding the distribution network within a microgrid improves the service reliability. Therefore, the size of the microgrids is determined by a trade-off between the controllability of microgrids and the reliability of service and network expansion within microgrids improve the reliability further.

Service restoration and reconfiguration performed by distribution management systems ensure the reliability of energy supply for the consumers; however, managing large-scale distribution automation systems with large number of DERs, switches, and controllable demands is a challenging task. Developing a robust real-time energy management framework with a large number of control variables is overwhelming for a central controller in the distribution network. Moreover, distributed energy management solutions to control the energy flow between the interconnected microgrids may not provide robust and reliable solutions [24]. Hence, sectionalizing the distribution networks, and islanding the microgrids will reduce the control processing burden and mitigates the disturbance and uncertainty propagation.

Allocating the generated energy to demand buses in each microgrid based on the ownership of the generation assets could be another criterion to determine the PCCs in the distribution networks. DER owner may offer to sell electricity to nearby demand buses within the microgrid [25]. The distribution network operator or distribution company may own the DERs in the system [26]-[27]. In such cases, the presented formulation could be used to assign certain consumers to the DERs and form islanded microgrids in contingencies within the distribution network.

The objective of this paper is to develop a framework for the expansion of the distribution network, while maximizing the number of autonomous microgrids to maintain the reliability and security of the energy supply. The procured PCCs determine the boundaries of the microgrids and disjoint the microgrids from the distribution network in contingencies. The expansion of the distribution lines improves the reliability of energy supply within microgrids. The main contributions of this paper are summarized as follows:

- A bi-level optimization problem is formulated to identify the PCCs by maximizing the number of microgrids considering the reliability requirements of the customers.
- The expansion of distribution lines ensures the reliability and security of energy supply in the island operation.
- Eigen decomposition of the Laplacian matrix of the distribution network graph is used to determine the graph spectra and formulate the connectivity of the distribution network graph.
- The uncertainties in demand and the availability of the components within the microgrids are captured by developing scenarios using the Monte Carlo simulation.

The rest of this paper is organized as the follows. In Section III, the application of Fielder theory to obtain the graph spectra of the distribution network and reconfigure the distribution network to form microgrids is presented. The problem formulation for the expansion of the distribution network to achieve the required reliability is discussed in Section IV. The effectiveness of the proposed framework is examined in multiple case studies in Section V. Finally; the conclusions are presented in Section VI.

III. Graph Spectra and Fiedler Theory

In this section, the spectra of the network are utilized to develop a formulation for sectionalizing the network [28]. The distribution network is represented as a graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$ as the buses and lines are presented by vertices $\mathbf{V} \in \mathbb{N}^{NB}$ and edges $\mathbf{E} \in \mathbb{N}^{NL}$ respectively. Graph $\mathbf{G}$ has a corresponding Laplacian matrix $\mathbf{L} \in \mathbb{Z}^{NB} \times \mathbb{Z}^{NB}$ in which the diagonal arrays are the total number of edges connecting to the vertices and the off-diagonal arrays represent the number of edges connecting vertices multiplied by "-1". The Laplacian matrix of a graph can be decomposed into its eigenvalues $\lambda_1 \le \lambda_2 \le \cdots \le \lambda_{NB}$ as shown by (1). Here, $\mathbf{Q}$ is the eigenvector matrix and $\mathbf{\Lambda}$ is the diagonal matrix of eigenvalues.

$$\mathbf{L} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^T \quad (1)$$

A sample graph is represented in Fig. 1, and the spectra of this graph which represent its connectivity are given in Fig. 2. For a connected undirected graph, the smallest eigenvalue of the Laplacian matrix is always zero and the second smallest eigenvalue, also called Fiedler eigenvalue, represents the algebraic connectivity of the graph [29]. The number of zero eigenvalues is equal to the number of isolated subgraphs in a graph [28]. In the graph shown in Fig. 1b, the first two eigenvalues are zero as shown in Fig. 2. One extreme case is the fully connected graph where the first eigenvalue will be zero and other eigenvalues are equal to the number of vertices in the graph. The other extreme case is the fully disconnected graph where all the eigenvalues are zero, so the number of sub-graphs is equal to the number of vertices in the graph.

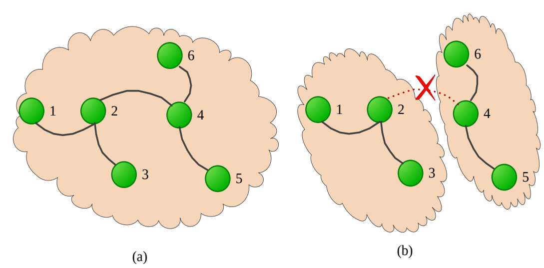

Fig. 1. The sample graph: a) Connected graph b) Sectioned graph

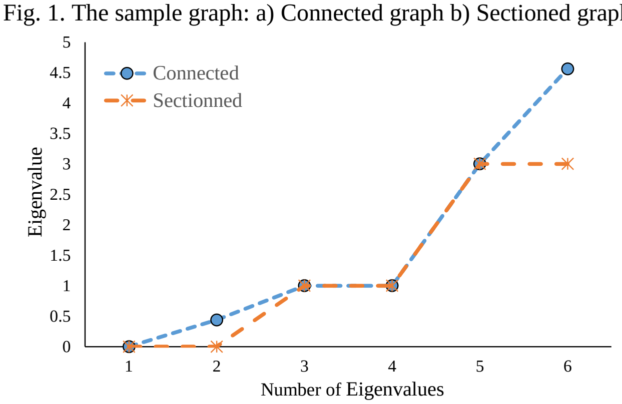

Fig. 2. The spectra of the sample graph

IV. Problem formulation

In this section, a new formulation is proposed to sectionalize the distribution network into microgrids and the Fielder eigenvalue is used as a measure to determine the number of sub-networks in a network. The problem is formulated as a bi-level mixed integer nonlinear problem (MINLP) [30] in which, the binary variables were associated with each edge of the distribution network graph i.e. distribution lines. If the binary variable is 1 the distribution line remains connected, otherwise, the line will be disconnected to form microgrid. The objective of the upper-level problem is to procure the maximum number of microgrids while the lower-level problem ensures the reliability of the formed microgrids. Therefore, in the upper-level problem, the distribution lines are disconnected to increase the number of islanded microgrids in the case of a disturbance in distribution network; however, the objective of the lower-level problem is to expand the distribution network within the formed microgrids to ensure that the required level of reliability at demand buses is satisfied considering limited budget for such expansions. Employing the duality theory, the presented bi-level problem is transformed into a single-level problem in which the lower-level problem is considered as a constraint for the upper-level problem [31]. The expansion decisions are further validated to avoid over-investing on the distribution assets by minimizing the cost of expansion plans considering the reliability constraints in the formed microgrids.

A. Network topology problem (Upper-level problem)

The upper-level problem minimizes the eigenvalues to maximize the number of microgrids in the distribution network. The eigenvalue minimization problem has a convex feasible set and can be represented as a semi-definite programming problem [32]. In the upper-level problem (2)-(8), the decision variables are the connection states of the distribution lines, which determine the PCCs in the distribution network. In order to determine the decision variables, the eigenvalues of the Laplacian matrix of the distribution network graph are minimized in an ascending order to ensure that $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_{NG}$. The sum of the weighted smallest eigenvalues – which are equal in number to the number of DERs – is minimized in (2). In an undirected graph, the sum of eigenvalues of the Laplacian matrix of the graph is equal to the sum of vertex degrees. In order to procure the maximum number of microgrids, the eigenvalues of the Laplacian matrix of the distribution network graph should be zero in an ascending order. The first eigenvalue is always zero. If the first two eigenvalues become zero, the distribution network will be divided into two microgrids. Similarly, if the first three eigenvalues become zero, the distribution network will be divided to three microgrids. Thus, the objective function of the upper-level problem is formulated to procure zero eigenvalues in an ascending order, and the number of formed microgrids cannot exceed the number of DERs. Here, the priority is to minimize the eigenvalues with lower orders. Minimizing the sum of the eigenvalues does not guarantee that the eigenvalues will be zero in an ascending order. Therefore, eigenvalues are weighted in the objective function of the upper-level problem.

Each eigenvalue is determined by multiplying the transposed matrix of eigenvectors, the Laplacian matrix, and the eigenvector matrix as shown in (3). Thus, minimizing the eigenvalues will change the Laplacian matrix, and consequently the topology of the distribution network. As indicated by (4), the eigenvectors corresponding to each eigenvalue of the Laplacian matrix are orthogonal. The eigenvector multiplication is a nonlinear but convex function. The eigenvector corresponding to each eigenvalue of the Laplacian matrix are normalized to unit length as shown in (5). In (6) and (7), the relationship between the Laplacian matrix of the distribution network and the decision variables associated with the connectivity of distribution lines (edges of the associated graph) are given. Here, $diag(L)$ represents the diagonal elements of the Laplacian matrix L. In (8), the investment for adding the new distribution lines is limited by the total investment budget (B), and $c_{b,b'}$ is the cost associated with adding a new distribution line. Therefore, for the existing distribution lines, this parameter is set to zero. As the upper-level problem minimizes the weighted eigenvalue spectra of the Laplacian matrix of the distribution network graph – which also represents the connectivity of the distribution network graph – it will limit the expansion of the distribution network as more distribution lines will increase the connectivity of the network graph. The expansion of the distribution network is further limited by the available budget for installing the distribution lines. It worth noting that other economic criteria such as ownership as well as the PCC and controller costs were not incorporated in the presented formulation. Such criteria could be addressed in the upper-level problem by adding constraints for the connectivity of certain vertices in the distribution network graph. Moreover, the costs associated with PCC and controllers could be incorporated in (8).

$$\min \sum_{b}^{NG} \left((NB+1) - b \right) \lambda_b \tag{2}$$

$$\lambda_b = \mathbf{q}_b^T \mathbf{L} \mathbf{q}_b \qquad b = 1, \ldots. NB \tag{3}$$

$$\mathbf{q}_b^T \mathbf{q}_{b'} = 0 \qquad b \neq b' \quad b, b' = 1, \ldots. NB \tag{4}$$

$$\mathbf{q}_b^T \mathbf{q}_{b'} = 1 \qquad b = b' \quad b, b' = 1, \ldots. NB \tag{5}$$

$$diag(\mathbf{L}) = \sum_b u_{b,b'} \qquad b \neq b' \quad b, b' = 1, \ldots. NB \tag{6}$$

$$l_{bb'} = -u_{b,b'} \qquad l_{bb'} \in \mathbf{L} \quad b \neq b' \quad b, b' = 1, \ldots. NB \tag{7}$$

$$\sum_b c_{b,b'} . u_{b,b'} \leq B \qquad b \neq b' \quad b, b' = 1, \ldots. NB \tag{8}$$

B. Reliability evaluation problem (Lower-level problem)

The lower-level problem minimizes the expected operation cost of the autonomous microgrids while maintaining the reliability requirements of the individual demands. The decision variables in the lower-level problem (9)-(24) are the real and reactive power dispatch of the DERs and the demand served at buses in the distribution network considering the determined topology in the upper-level problem. Here, the decisions made on the connectivity of the distribution lines in the upper-level problem are constrained by the reliability requirements at demand buses captured in the lower-level problem. This problem is presented as linear programming (LP) problem. The objective function is the expected operation cost of the autonomous microgrids which includes the operation cost of generating electricity and purchasing electricity from the main grid, as well as the penalty associated with demand curtailment as shown in (9). The real and reactive power injection at each bus of the distribution network is shown by (10) and (11), respectively. At each bus, the available generation is the summation of the power generated by DER and the imported power from the utility, if the bus is connected to the feeder. The real and reactive power limitations of DERs are shown in (12) and (13) respectively. The expected energy not supplied (EENS) for real and reactive demand at each bus is limited by (14) and (15) respectively. The EENS for the demand at each bus is procured by multiplying the loss of energy probability (LOEP) and the total expected energy consumed. As shown in (16), the admittance of distribution line incorporates the binary variable representing its connectivity, which is determined by the upper-level problem, as well as the binary parameter representing the line outage in different scenarios. The real and reactive power injection at each bus is linearized in (17) and (18). The constraints (19), (20) and (21) present the linearized formulation for real, reactive and apparent power transmitted through the distribution line [33]. The power flow in the distribution line is limited by the line capacity as given in (22). The voltage magnitude and phase angle at each bus in the distribution network are limited by (23) and (24), respectively.

$$\min_{P_i^{t,s},P_g^{t,s}} \sum_{s}^{NS}\sum_{t}^{NT}\sigma_t\rho^s\begin{pmatrix}\sum_i\left(F_{c,i}(P_i^{t,s})\right)+\sum_g\left(c_g^{t,s}P_g^{t,s}\right)\\ +VOLL_b\left(P_{b,t}^{D,s}-P_{b,t}^{d,s}\right)\end{pmatrix} \tag{9}$$

s.t.

$$\sum_{g\in GR_b} UZ_g^{t,s}.P_g^{t,s}+\sum_{i\in GG_b}P_i^{t,s}-P_{b,t}^{d,s}=P_{b,t}^{inj,s} \tag{10}$$

$$\sum_{g\in GR_b} UZ_g^{t,s}.Q_g^{t,s}+\sum_{i\in GG_b}Q_i^{t,s}-Q_{b,t}^{d,s}=Q_{b,t}^{inj,s} \tag{11}$$

$$P_i^{\min}\cdot UX_i^{t,s}\le P_i^{t,s}\le P_i^{\max}.UX_i^{t,s} \tag{12}$$

$$Q_i^{\min}\cdot UX_i^{t,s}\le Q_i^{t,s}\le Q_i^{\max}\cdot UX_i^{t,s} \tag{13}$$

$$0\le\sum_{t}^{NT}\sum_{s}^{NS}\sigma_t\rho^s\left(P_{b,t}^{D,s}-P_{b,t}^{d,s}\right)\le EENS_b \tag{14}$$

$$0\le Q_{b,t}^{d,s}\le Q_{b,t}^{D,s} \tag{15}$$

$$y_{b,b'}^{t,s}=g_{b,b'}^{t,s}+Jb_{b,b'}^{t,s}=\frac{r_{b,b'}.UY_{b,b'}^{t,s}.u_{b,b'}}{r_{b,b'}^2+x_{b,b'}^2}-J\frac{x_{b,b'}.UY_{b,b'}^{t,s}.u_{b,b'}}{r_{b,b'}^2+x_{b,b'}^2} \tag{16}$$

$$\begin{aligned}P_{b,t}^{inj,s}&=\left(2V_b^{t,s}-1\right)G_{b,b}^{t,s}+\\&\sum_{b'(b'\ne b)}^{NB}\left(G_{b,b'}^{t,s}\left(V_b^{t,s}+V_{b'}^{t,s}-1\right)+B_{b,b'}^{t,s}\left(\theta_b^{t,s}-\theta_{b'}^{t,s}\right)\right)\end{aligned} \tag{17}$$

$$\begin{aligned}Q_{b,t}^{inj,s}&=-\left(2V_b^{t,s}-1\right)B_{b,b}^{t,s}+\\&\sum_{b'(b'\ne b)}^{NB}\left(-B_{b,b'}^{t,s}\left(V_b^{t,s}+V_{b'}^{t,s}-1\right)+G_{b,b'}^{t,s}\left(\theta_b^{t,s}-\theta_{b'}^{t,s}\right)\right)\end{aligned} \tag{18}$$

$$PL_{b,b'}^{t,s}=-G_{b,b'}^{t,s}(V_b^{t,s}-V_{b'}^{t,s})+B_{b,b'}^{t,s}(\theta_b^{t,s}-\theta_{b'}^{t,s}) \tag{19}$$

$$QL_{b,b'}^{t,s}=B_{b,b'}^{t,s}(V_b^{t,s}-V_{b'}^{t,s})+G_{b,b'}^{t,s}(\theta_b^{t,s}-\theta_{b'}^{t,s}) \tag{20}$$

$$SL_{b,b'}^{t,s}=PL_{b,b'}^{t,s}+\xi_{b,b'}QL_{b,b'}^{t,s} \tag{21}$$

$$\left|SL_{b,b'}^{t,s}\right|\le SL_{b,b'}^{\max} \tag{22}$$

$$V^{\min}\le V_b^{t,s}\le V^{\max} \tag{23}$$

$$\theta^{\min}\le\theta_b^{t,s}\le\theta^{\max} \tag{24}$$

C. Microgrid topology validation problem

Minimizing the sum of the eigenvalues that are equal to in number to the number of DERs will not minimize the connectivity inside the formed microgrids. As the only constraint for limiting the number of installed distribution lines is the limitation on the total budget, unnecessary distribution lines may be installed. Installing the distribution lines will improve the reliability in the microgrids, however, such decisions may not be economical once the reliability levels requested by demand entities are satisfied. To address this issue, a microgrid topology validation problem is proposed to check if the added distribution lines are necessary to satisfy the reliability requirements. Eliminating the unnecessary distribution lines within the formed microgrids will not change the number of zero eigenvalues determined in the bi-level optimization problem and consequently will not change the boundary of the formed microgrids. By removing the unnecessary distribution lines within the microgrids, the sum of the non-zero eigenvalues is decreased. Therefore, the algebraic connectivity is decreased within microgrids while the reliability requirements are satisfied. The objective of the microgrid topology validation problem is given in (25), in which the investment cost of adding new distribution lines and the operation cost of the system are minimized. As this problem checks for the unnecessary installation of the distribution lines, the decision variables in this problem are limited by the procured decisions ($u'_{b,b'}$) of the bi-level optimization problem (2)-(24) as shown in (26). This means that this problem seeks to validate the decisions made in the bi-level problem. The objective function for this problem is subjected to set of constraints (10)-(24).

$$\min_{P_i^{t,s},P_g^{t,s},u_{b,b'}}\sum_{s}^{NS}\sum_{t}^{NT}\sigma_t\rho^s\begin{pmatrix}\sum_i\left(F_{c,i}(P_i^{t,s})\right)+\sum_g\left(c_g^{t,s}P_g^{t,s}\right)\\ +VOLL_b\left(P_{b,t}^{D,s}-P_{b,t}^{d,s}\right)\end{pmatrix}+\sum_b\sum_{b'\ne b}c_{b,b'}.u_{b,b'} \tag{25}$$

$$u_{b,b'}\le u'_{b,b'}\qquad \forall b,b',b'\ne b \tag{26}$$

D. Solution Methodology

The presented bi-level MINLP problem (2)-(24) is expressed in the general form by (27)-(32). By employing the duality theory, the presented problem is transformed into a single-level problem (33)-(40). The other approach for such transformation is to use the Karush Kuhn Tucker (KKT) conditions for the lower-level problem; however, using the duality theory will generate less number of the binary-to-continuous variable multiplications, which reduces the computation complexity and solution time. The decision variables in the upper-level problem include the connectivity I and the eigenvalue and eigenvectors $\boldsymbol{\eta}$. Here, (27) minimizes the weighted eigenvalues in an ascending order as shown in (2). The binary variables are the decision vectors representing the connectivity of distribution network including the decisions on the location of PCCs and the installation of new distribution lines and the continuous variables are the eigenvalue and eigenvectors. Set of constraints (28) represent the equality constraints in the upper-level problem including the equality constraint from which the eigenvalues of the Laplacian matrix are determined (3), the properties of the eigenvectors (4) and (5), and the relationship between the connectivity of the distribution network graph and its Laplacian matrix (6), (7). Constraint (8), which represents the limited budget for installing new distribution lines, is presented by (29). The lower-level problem is shown in (30)-(32) in which the objective function (9) captures the binary variables and re-written as (30), where vector **d** represents the parameters in the objective of the lower-level problem. The set of equality constraints (31) represent the equality constraints (10), (11), (16)-(21). Similarly, the set of inequality constraints (32) represents the inequality constraints (12)-(15), and (22)-(24). The variables in this problem include the decision variables correspond to connectivity represented by $\mathbf{I}$, the eigenvalues and eigenvectors represented by $\boldsymbol{\eta}$, the continuous variables in the lower-level problem represented by $\boldsymbol{\alpha}$, and the Lagrange multipliers of the equality and inequality constraints in the lower-level problem represented by $\boldsymbol{\pi}$ and $\boldsymbol{\tau}$.

$$\min_{\mathbf{I},\boldsymbol{\eta}} \gamma(\boldsymbol{\eta}) \tag{27}$$

s.t.

$$g(\mathbf{I},\boldsymbol{\eta}) = \mathbf{K} \qquad \mathbf{I} \in \{0,1\} \tag{28}$$

$$f(\mathbf{I}) \le \mathrm{E} \tag{29}$$

$$\min_{\boldsymbol{\alpha}} \mathbf{d}^T \boldsymbol{\alpha} \tag{30}$$

s.t.

$$A(\mathbf{I}) + B\boldsymbol{\alpha} = \mathbf{b_1} \qquad (\boldsymbol{\pi}) \tag{31}$$

$$C(\mathbf{I}) + D\boldsymbol{\alpha} \le \mathbf{b_2} \qquad (\boldsymbol{\tau}) \tag{32}$$

By employing the duality theory, the presented problem in (27)-(32) is transformed into a single-level problem as shown in (33)-(40), where the lower-level problem is considered as a set of constraints for the upper-level problem [29]. The objective is the same as the objective of the upper-level problem, which minimizes the eigenvalues of the distribution network graph in an ascending order. Here, the objective and constraints of the upper-level problem are shown in (33)-(35) and the lower-level problem (30)-(32) is shown as a set of constraints (36)-(40). The constraints (36) and (37) are the constraints of the primal lower-level problem, while (38) and (39) are the constraints correspond to the dual representation of the lower-level problem. The strong duality condition, which is shown in (40), holds once the lower-level problem is linear and therefore the duality gap is zero. Here, the objective of the primal lower-level problem is equal to the objective of the dual lower-level problem. The binary to continuous terms in (40) are further linearized by (41)-(44).

$$\min_{\mathbf{I},\boldsymbol{\eta},\boldsymbol{\alpha},\boldsymbol{\pi},\boldsymbol{\tau}} \gamma(\boldsymbol{\eta}) \tag{33}$$

s.t.

$$g(\mathbf{I},\boldsymbol{\eta}) = \mathbf{K} \qquad \mathbf{I} \in \{0,1\} \tag{34}$$

$$f(\mathbf{I}) \le \mathrm{E} \tag{35}$$

$$A(\mathbf{I}) + B\boldsymbol{\alpha} = \mathbf{b_1} \tag{36}$$

$$C(\mathbf{I}) + D\boldsymbol{\alpha} \le \mathbf{b_2} \tag{37}$$

$$B^T \boldsymbol{\pi} + D^T \boldsymbol{\tau} = \mathbf{d} \tag{38}$$

$$\boldsymbol{\tau} \ge \mathbf{0} \tag{39}$$

$$\mathbf{d}^T \boldsymbol{\alpha} = \boldsymbol{\pi}^T [\mathbf{b_1} - A(\mathbf{I})] - \boldsymbol{\tau}^T [\mathbf{b_2} - C(\mathbf{I})] \tag{40}$$

The equivalent single-level problem has several binary-to-continuous nonlinear terms. Such terms appear in the AC power flow formulation and in the dual form of the lower problem. In (17)-(20) the binary decision variables for the connectivity of the distribution network and installation of the distribution lines are integrated into the admittance matrix as shown in (16).

The nonlinear binary-to-continuous terms illustrated in (41) are linearized by presenting the constraints (42)-(44). Two auxiliary nonnegative variables, Φ and Ψ are employed and M is an arbitrary large number.

$$\Psi = \mathrm{I}\,\tau, \quad \mathrm{I} \in \{0,1\} \tag{41}$$

$$\Psi + \Phi = \tau \tag{42}$$

$$0 \le \Psi \le M \cdot \mathrm{I} \tag{43}$$

$$0 \le \Phi \le M \cdot (1 - \mathrm{I}) \tag{44}$$

The other sources of nonlinearities are the constraints that determine the eigenvalues. These constraints form convex feasibility sets and an optimal solution of the presented MINLP is procured using convex optimization approaches. Several approaches may be employed to solve the presented convex MINLP problem. One of these methods is generating and successively improving the outer approximations in the neighborhood of a set of optimal solutions for the MINLP. By utilizing the linear outer-approximation and introducing auxiliary variables, the MINLP is reformulated into MIP problem that could be solved by branch and bound [34] or cutting plane techniques [35]. Therefore, the presented problem could be solved utilizing solvers (e.g. Alpha-ECP, BARON, and SCIP) that employ similar methodologies [36]-[40]. Finally, the decisions for adding new distribution lines will be validated in the microgrid topology validation problem. This problem is formulated as a MIP problem in which the decision variables are the installation states of the distribution lines. The objective of this problem is to minimize the installation cost of the distribution lines and the expected operation cost of the microgrids subjected to generation and distribution network constraints and the reliability requirements of the demands.

V. Case Study

a) A sample meshed distribution system

In this section, an electricity distribution network, which is composed of 20 buses, is illustrated in Fig. 3. The presented distribution network is normally connected to the utility feeder while it is divided into multiple microgrids fed by 5 DERs in the case of any disruption or disturbance in the distribution network. The distribution network has 23 existing electricity distribution lines, and 20 electricity demands. Nine electricity distribution lines are considered as candidates in the distribution network. Table I presents the characteristics of the

DERs and Table II shows the associated loss of energy probability (LOEP) [41]-[44] at each bus which determines the priority of demands served in the distribution network and calculated by (45). Table III describes the characteristics of the electricity distribution lines including the forced outage rates (FOR), buses that are connected by each line, length and the maximum capacity of the lines. The FOR associated with the feeder is 10.9 f/yr and the price of electricity at the feeder is 8 ¢/kWh. Here, it is assumed that the available budget is sufficient for 1,000-meter installation of the distribution lines. The states of the components are calculated based on the availability of the components using two state Markov Chain process and Monte Carlo simulation. Here, 3000 scenarios are generated to represent the uncertainties in the system; including the generator and distribution line random outages, and the demand uncertainties. Truncated normal distribution function is used to represent the errors in load forecast, where the mean value is the forecasted volume and the standard deviation is percentages of the mean values [45], [46]. Here, the standard deviation is 5% from the mean value. The load profile is represented by the load duration curve which is divided into four periods $\sigma_1, \sigma_2, \sigma_3$, and σ_4 with 10%, 25%, 60%, and 5% of the total annual hours. The scenario reduction technique [47]-[48] are employed to reduce 3000 scenarios to 12 scenarios. The voltage magnitude and phase angles of buses are restricted between 0.95-1.05 per unit and (-π) to (π) respectively. The resistance and inductive reactance of the distribution network cables are $r = 0.092\Omega/1000m$ and $x = 0.121\Omega/1000m$ respectively. The equivalent single level MINLP is solved by SCIP 3.2.0 that employs branch and bound algorithm, wherein the relaxed LP is tightened by cutting plane and domain propagation. In order to handle the computation burden, sets of distribution lines that satisfy primal dual representation of the reliability evaluation problem are procured and the set with minimum Eigenvalues is selected using SCIP solver.

$$LOEP_b = EENS_b \Big/ \sum_t \sum_s \sigma_t \rho^s P_{b,t}^{D,s} \quad (45)$$

$$VOLL_b = 1/LOEP_b \quad (46)$$

TABLE I
POWER GENERATION UNIT CHARACTERISTICS

Unit	Pmax[1] (kW)	Pmax[2] (kW)	F_c^1 (¢/kWh)	F_c^2 (¢/kWh)	P_{max} (kW)	Q_{max} (kvar)	Q_{min} (kvar)	FOR (f/yr)
G1	1,200	600	8	15	1,800	900	-900	3.65
G2	900	450	10	28	1,350	625	-625	14.6
G3	1,000	500	10	24	1,500	750	-750	7.3
G4	900	450	9	14	1,350	600	-600	10.9
G5	750	325	12	25	1,075	550	-550	7.3

TABLE II
REQUIRED LOEP AT EACH BUS

Bus	$LOEP_b$	Bus	$LOEP_b$	Bus	$LOEP_b$	Bus	$LOEP_b$
1	0.03	6	0.005	11	0.05	16	0.03
2	0.005	7	0.001	12	0.09	17	0.07
3	0.03	8	0.001	13	0.05	18	0.01
4	0.01	9	0.07	14	0.08	19	0.02
5	0.02	10	0.04	15	0.05	20	0.03

Table IV shows the reliability indices at each bus achieved in this configuration with no microgrids or DERs. As shown here, the LOEPs of the loads are not within the required range and the LOEPs exceed the acceptable level at buses 2, 4, 6, 7, 8, 11, 14, and 18. Considering the value of lost load as defined in (46) in $/kWh, the expected annual operation cost of the distribution network is $1.773M. The total expected demand and the total expected demand curtailment are 17,361 MWh and 2,247 MWh respectively. The following two cases are considered,

1) Case 1 – heterogeneous LOEP requirement for demands
2) Case 2 – non-heterogeneous LOEP requirement for demands

TABLE III
DISTRIBUTION LINES CHARACTERISTICS

ID	FOR (f/yr)	Length (m)	SL^{max} (kVA)	ID	FOR (f/yr)	Length (m)	SL^{max} (kVA)
1	3.65	220	800	17	7.3	90	800
2	7.3	140	1,200	18	3.65	440	1,700
3	3.65	90	800	19	3.65	280	1,200
4	7.3	210	880	20	7.3	260	1,100
5	3.65	120	1,000	21	3.65	200	1,200
6	3.65	80	800	22	3.65	300	1,200
7	3.65	140	1,700	23	7.3	210	900
8	7.3	270	1,200	24	3.65	190	900
9	7.3	350	1,200	25	3.65	460	2,000
10	3.65	200	1,200	26	3.65	260	800
11	3.65	300	1,200	27	7.3	500	1000
12	3.65	120	1,200	28	7.3	450	900
13	7.3	110	1,200	29	3.65	390	900
14	3.65	160	2,000	30	7.3	360	1,000
15	7.3	220	800	31	3.65	500	800
16	7.3	120	1,000	32	3.65	460	1,000

TABLE IV
THE LOEP IN DISTRIBUTION NETWORK WITH NO DER AND MICROGRID

Bus	$LOEP_b$	Bus	$LOEP_b$	Bus	$LOEP_b$	Bus	$LOEP_b$
1	0.015	6	0.015	11	0.342	16	0.015
2	0.015	7	0.015	12	0.022	17	0.019
3	0.022	8	0.015	13	0.019	18	0.015
4	0.015	9	0.019	14	1	19	0.015
5	0.02	10	0.019	15	0.015	20	0.015

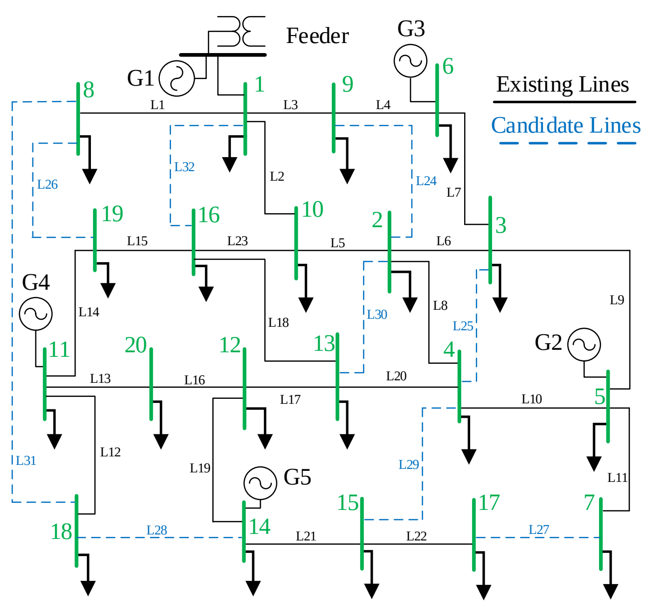

Fig. 3. The proposed electricity distribution network

A. *Case1: Heterogeneous LOEP requirement for demands*

In this case, the values for the required LOEP for demands at each bus are given in Table II. The spectra of the graph associated with the distribution network are shown in Fig. 4.

The first five eigenvalues are minimized and the remaining eigenvalues are determined by validating the microgrid topology. In this case, as the 5 smallest eigenvalues of the corresponding Laplacian matrix are zero, placing the breakers at PCCs would help to divide the distribution network into five autonomous microgrids as shown in Fig. 5. Here, microgrid M1 is composed of buses {1, 2, 3, 8, 9, 10, 13, 16} which is supplied by the G1. Microgrids M2, M3, M4, and M5 are composed of buses {5, 7}, {6}, {4, 12, 14, 15, 17}, and {11, 18, 19, 20} which are respectively supplied by G2, G3, G5, and G4. The PCCs for microgrids M1 and M3 are located on lines L4 and L7. The PCC between microgrids M1 and M2 is located on lines L9 and the PCC between microgrids M2 and M4 is located on line L10. Moreover, the PCCs between microgrids M1 and M4 are located on lines L8, L17, and L20. The PCCs between microgrids M5 and M1; and between microgrids M5 and M4; are located on lines L15 and L16, respectively. To address the reliability requirements, distribution lines L32 and L29 with a total length of 850 meters were installed in the existing distribution network. Table V shows the reliability indices achieved in this configuration. As shown in this table, the LOEPs of the loads are within the required range, while the LOEP reached but not exceeded the maximum acceptable level at bus 4. Considering the value of lost load as defined in (46) in $/kWh, the expected annual operation cost of the distribution network with microgrids is $1.682M which is 2.93% less than the operation cost of the distribution network in normal condition ($1.733M). Although more expensive DER units serve demands within each microgrid, the reduction in the demand curtailment reduces the operation cost. Hence, the formation of microgrids with DERs improves the operation cost of the distribution network while the PCCs will form microgrids in the event of disruptions or failures in the distribution network.

TABLE V
THE LOEP OF DEMAND AT EACH BUS IN CASE 1

Bus	$LOEP_b$	Bus	$LOEP_b$	Bus	$LOEP_b$	Bus	$LOEP_b$
1	0.0006	6	0.002	11	0.003	16	0.0006
2	0.0006	7	0.00005	12	0.09	17	0.016
3	0.0006	8	0.0006	13	0.0006	18	0.004
4	0.01	9	0.0006	14	0.008	19	0.003
5	0.00005	10	0.0006	15	0.008	20	0.003

The presented framework provides the necessary tool to define boundaries of each microgrid. The procured microgrids are able to satisfy the reliability requirements of the demands in island mode. The total expected demand curtailment is 112 MWh while the total expected annual demand is 17,361 MWh. Forming the microgrids would assign the DERs to serve certain demands by islanding which leads to heterogeneous LOEPs in the distribution network. Hence the presented approach ensures the reliability requirements of the demands within the distribution network, while maximizing the number of microgrids in the distribution network.

B. Case 2: Non-heterogeneous LOEP requirement for demands

In this case, the new topology of the distribution network is procured considering a uniform required LOEP for the demands. The required LOEP is the lowest required LOEP shown in Table II. The spectra of the distribution network graph are shown in Fig. 4. Since the 3 smallest eigenvalues of the related Laplacian matrix are zero, the distribution network is sectionalized into 3 autonomous microgrids as shown in Fig. 6. Here, microgrid M1 is composed of buses {1, 2, 3, 4, 6, 8, 9, 10, 13, 15, 16, 17} which are supplied by the G1 and G3. Microgrids M2 and M3 are composed of {5, 7}, {11, 12, 14, 18, 19, 20} which are respectively supplied by G2; G4 and G5. The PCC between microgrids M1 and M2 are placed on lines L9 and L10. In addition, PCCs between microgrids M1 and M3 are located on lines L15, L17, and L21. To address the reliability requirements, the distribution lines L28 and L29 with a total length of 860 meters were installed in the existing distribution network. The required reliability for all demand entities is ensured as shown in Table VI. Here the required LOEPs for buses 5 and 7 are the same as those in Case 1 because the topology of microgrid M2 in this case, is the same as microgrid M2 in Case 1. With the value of loss load defined in (46) in $/kWh, the expected cost of the distribution network is $1.492M in this case which is 11.24% less than that in Case1. Compared to Case 1, some of the demands require lower LOEP in Case 2 which leads to larger microgrids (less zero eigenvalues for the graph spectra that indicates more interconnected distribution network) as shown in Fig. 4. The total expected demand curtailment is 709.56 kWh. The LOEP for each demand is calculated in Table VI.

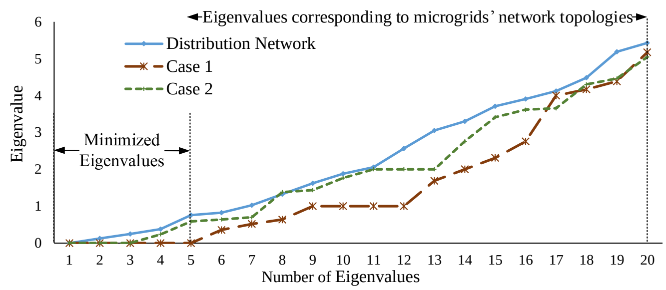

Fig. 4. The spectra of the graph associated with distribution network

TABLE VI
THE LOEP OF DEMAND AT EACH BUS IN CASE 2

Bus	$LOEP_b$	Bus	$LOEP_b$	Bus	$LOEP_b$	Bus	$LOEP_b$
1	0	6	0	11	0	16	0
2	0.0001	7	0.00005	12	0.0006	17	0.00009
3	0	8	0	13	0	18	0
4	0	9	0	14	0	19	0
5	0.00005	10	0	15	0.00009	20	0

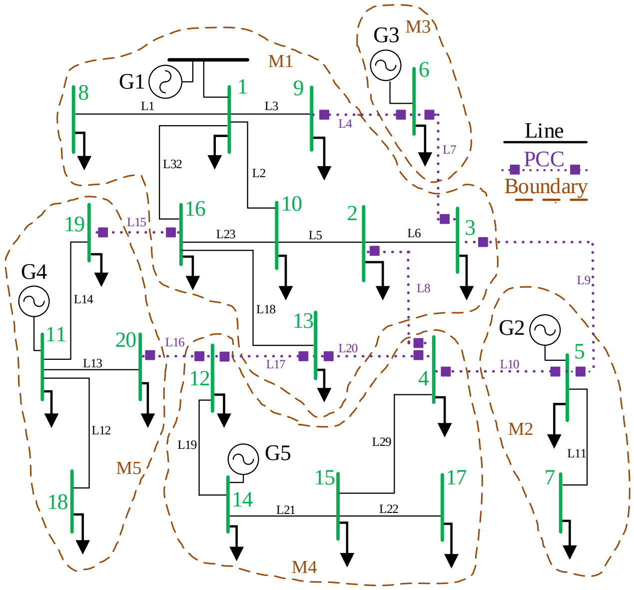

Fig. 5. Electricity distribution network in Case 1

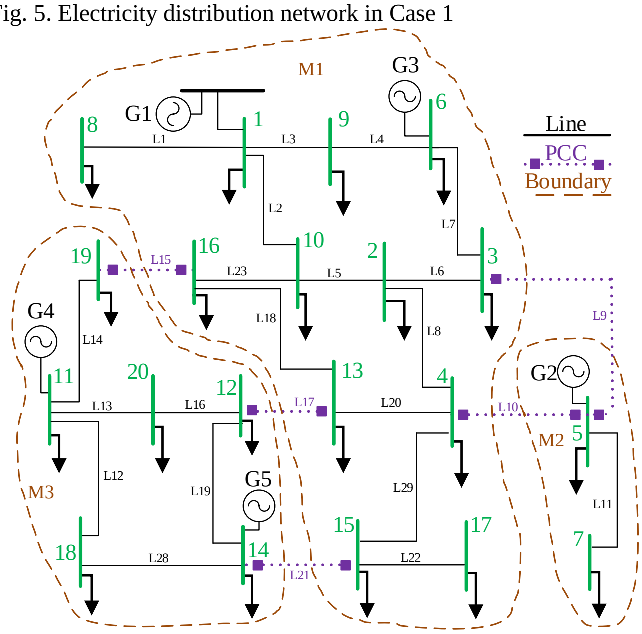

Fig. 6. Electricity distribution network in Case 2

b) The Modified IEEE 123-bus system

The IEEE 123-bus system [48] is fed by the utility feeder in normal condition as shown in Fig. 7. The network will sectionalize into several microgrids served by 10 DERs in the case of disturbances in the network. Here, 8 distribution lines are considered as candidates in the distribution network to satisfy the required LOEP at the demand buses. The required LOEPs at demand buses are given in Table VII. As the 9 smallest eigenvalues of the corresponding Laplacian matrix of the distribution network graph are zero, the distribution network is sectionalized into 9 autonomous microgrids. Microgrid M1 is composed of buses {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 34, 149} which are supplied by the DER on bus 7 with the FOR of 10.9 f/yr. Microgrid M2 is composed of buses {18, 19, 20, 21, 22, 23, 24, 35, 36, 37, 38, 39, 135} which are supplied by the DER on bus 21 with the FOR of 3.65 f/yr. The tie switch between buses 18 and 135 is closed and the PCC between microgrids M1 and M2 is placed on line L13, which connects buses 13 and 18. Microgrid M3 is composed of buses {25, 26, 27, 28, 29, 30, 31, 32, 33, 250} which are supplied by the DER on bus 26 with the FOR of 7.3 f/yr. Here, the PCC between microgrids M2 and M3 is placed on line L24 that connects buses 23 and 25. Microgrid M4 is composed of buses {40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 151} which are supplied by the DER on bus 44 with the FOR of 10.9 f/yr. Here, the tie switch between buses 151 and 300 is open and the PCC between microgrids M2 and M4 is placed on line L36 that connects the buses 35 and 40. Microgrid M5 is the largest designated microgrid that is composed of buses {52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 67, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 152, 160, 610} which are supplied by the DERs on buses 53 and 76 with the FOR of 10.9 f/yr and 14.55 f/yr respectively. Here, the tie switch between buses 60 and 160 is closed and the PCC between microgrids M1 and M5 is located at the tie switch that connects buses 152 and 13. Microgrid M6 is the smallest designated microgrid that is composed of buses {62, 63, 64, 65, 66} which are supplied by the DER on bus 62 with the FOR of 3.65 f/yr. Here, the PCC between microgrids M6 and M5 is on line L61 that connects buses 60 and 62. Microgrid M7 is composed of buses {87, 88, 89, 90, 91, 92, 93, 94, 95, 96} which are supplied by the DER on bus 89 with FOR of 7.3 f/yr. Here, the PCCs between microgrids M5 and M6 are on line L86 that connects buses 86 and 87, and on the tie switch that connects buses 54 and 94. Microgrid M8 is composed of buses {68, 69, 70, 71, 97, 98, 99, 100, 450} which are supplied by the DER on bus 98 with the FOR of 3.65 f/yr. To satisfy the reliability requirements at the load point on bus 69, a 200-meter distribution line connects buses 70 and 100. The PCCs between microgrids M5 and M8 are located on lines L66 and L68 that connect bus 67 to buses 68 and 97 respectively.

The FOR of the line L69 that connects buses 68 and 69 is 7.3 f/yr, which leads to multiple outages in the operation horizon. Such high FOR results in unacceptable LOEP of the demand on bus 69 (0.06) before forming microgrid M8. The required LOEP for the demand on buses 68 and 69 are 0.06236 and 0.0078 respectively. The network expansion in microgrid M8 by installing the distribution line that connects buses 70 and 100, will further reduce the LOEP of demand on bus 69 from 0.06 to zero, and the LOEP of demand on bus 68 from 0.349 to 0.059. Thus, network expansion within autonomous microgrids further improves the reliability of supplied energy.

Finally, microgrid M9 is composed of buses {101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 197, 300} which are supplied by the DER on bus 108 with the FOR of 7.3 f/yr. Here, the PCC between microgrids M8 and M9 is located on the tie switch that connects buses 197 and 97, and the PCC between microgrids M4 and M9 is located on the tie switch that connects buses 151 and 300.

By considering 3.5% annual increase in demand in the 10-year operation horizon, and the value of lost load ($/kWh) shown in (46), the expected operation cost of the distribution network with microgrids is $45.6M. To illustrate the merit of forming microgrids in contingencies, the presented approach is compared with the network reconfiguration technique, which is discussed in [50]. Here, the existing tie-switches within the IEEE 123 bus test system are utilized to reconfigure the distribution network considering the outages in the distribution lines. The operation cost of the system with network reconfiguration capability is $50.9M, which is 11.7% more than the operation cost of the distribution network with microgrids. Similar to the first case study, forming microgrids supplied by DER units, enhances the reliability and reduces the operation cost of the distribution network.

TABLE VII
REQUIRED LOEP OF DEMANDS IN IEEE-123 BUS SYSTEM

Bus	$LOEP_b$	Bus	$LOEP_b$	Bus	$LOEP_b$	Bus	$LOEP_b$
1	0.03332	32	0.00516	63	0.01042	94	0.00104
2	0.11646	33	0.00984	64	0.0037	95	0.00516
3	0.2076	34	0.07228	65	0.00262	96	0.12146
4	0.11782	35	0.00056	66	0.01136	97	0.00084
5	0.0329	36	0.00974	67	0.00012	98	0.02432
6	0.24462	37	0.00042	68	0.06236	99	0.0652
7	0.06376	38	0.0002	69	0.0078	100	0.02492
8	0.02628	39	0.00868	70	0.000742	101	0.0023
9	0.00134	40	0.02484	71	0.00624	102	0.0012
10	0.01234	41	0.00364	72	0.00354	103	0.021
11	0.05062	42	0.00952	73	0.00274	104	0.00004
12	0.07316	43	0.00004	74	0.00222	105	0.04598
13	0.01622	44	0.00176	75	0.0611	106	0.00002
14	0.0724	45	0.00242	76	0.00034	107	0.08878
15	0.04926	46	0.08068	77	0.04038	108	0.03392
16	0.00112	47	0.01546	78	0.0006	109	0.00076
17	0.00056	48	0.00892	79	0.00096	110	0.00598
18	0.06462	49	0.06266	80	0.00104	111	0.09222
19	0.00006	50	0.05122	81	0.00006	112	0.00512
20	0.00014	51	0.05926	82	0.00302	113	0.000006
21	0.0024	52	0.00666	83	0.02186	114	0.1334
22	0.00012	53	0.02476	84	0.04486	197	0.02056
23	0.00056	54	0.0151	85	0.05452	160	0.00116
24	0.00182	55	0.02636	86	0.10082	152	0.01408
25	0.00008	56	0.0066	87	0.00338	149	0.00106
26	0.00034	57	0.00056	88	0.01694	135	0.00107
27	0.05124	58	0.003	89	0.00014	300	0.00003
28	0.00034	59	0.0147	90	0.000364	450	0.00001
29	0.00038	60	0.00134	91	0.01478	350	0.00055
30	0.03862	61	0.0372	92	0.11858	250	0.00016
31	0.00454	62	0.00946	93	0.0014		

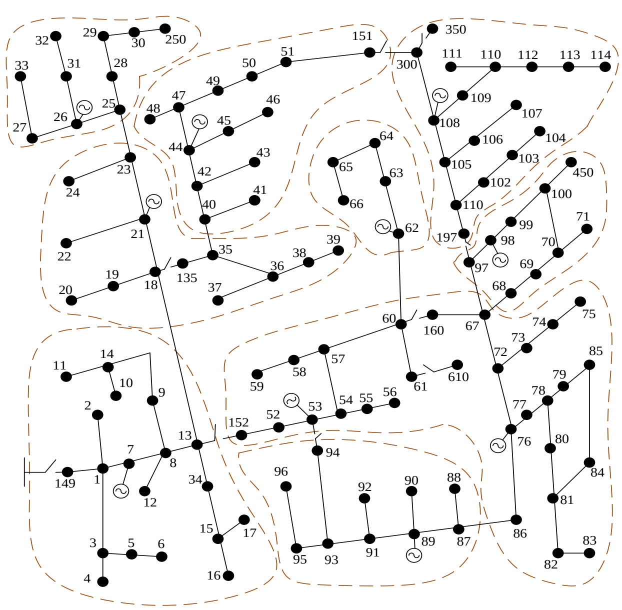

Fig. 7. Modified IEEE 123-bus test system

VI. CONCLUSION

This paper proposes a framework to sectionalize the distribution network into autonomous microgrids, considering heterogeneous reliability for the demands. The expansion of distribution network and formation of microgrids satisfy the reliability requirements in case of disruptions in the distribution network. The presented problem is formulated as a bi-level optimization problem. In the upper-level problem, the number of microgrids is maximized by minimizing the weighted sum of the eigenvalues of the Laplacian matrix of the distribution network graph. In the lower-level problem, the expected operation cost is minimized considering the reliability constraints for the demands at each bus. The procured microgrid topologies are further validated by minimizing the expansion and operation cost within the microgrids. The presented approach is evaluated by two case studies.

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