Optimal Switch Placement in Distribution Systems: A High-Accuracy MILP Formulation

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***Abstract*—A new solution method is introduced to the problem of optimally deploying manual and automatic switches in distribution systems, where the product of two continuous variables and the inverse of a continuous variable are reformulated as a linear relation. This leads to a (mixed integer linear problem) MILP power flow formulation too. The objective function includes cost and reliability. The cost term itself includes capital investment, installation, and maintenance costs (MC) as well as customer interruption cost (CIC); while the reliability term is represented by system average interruption duration index (SAIDI). The problem is formulated as a MILP, which guarantees a global optimal solution. The effectiveness of the proposed method is validated through various case studies and sensitivity analyses on the RBTS4, followed by a comprehensive discussion and analysis of results. The proposed MILP formulation prescribes fewer switches while achieving lower SAIDI, compared to that of a previous MINLP formulation.**

***Index Terms*— distribution automation system, reliability evaluations, switch placement, linear Power flow, Product of Two Continuous Variable, mixed integer linear programming.**

NOTATION

Sets:

$n_{t/c/f/k/b}$	Set of planning horizons/system load points/system faults/candidate location for installing switches/the maximum number of binary digits required to represent X_1.
N_i	Set of customers of load point i.
S	Set of switches.

Constants:

$C_{RCS/MS}^{Inv/M}$	Investment/Maintenance cost of automatic/manual switches.
Inf/Int/NInt	Inflation/Interest/Nominal interest rate.
r_j	Duration of fault j.
$P^{RCS/MS}$	Probability of automatic/manual switching.
$t^{RCS/MS}$	Automatic/Manual switching action time.
Budget	Investment cost capacity.
C^{min}	Minimum value of the total cost when Ψ=1.
$SAIDI^{min}$	Minimum value of the SAIDI reliability index when Ψ=0.

Variables:

$N^{RCS/MS}$	Total number of automatic/ manual switches.
I_{iw}	Current flow between i^{th} and w^{th} buses.
S_i	Apparent power of the i^{th} bus.
$I_{ij}^{RCS/MS}$	Binary variable that is equal to 1 if automatic/manual switch is existed in path of the fault j for load point i.
$I_k^{RCS/MS-B/E}$	Binary variable that is equal to 1 if manual/automatic switch is existed in the beginning/end of the line k and 0 otherwise.
$I_{ij}^{RCS/MS-B/E}$	Binary variable that is equal to 1 if manual/automatic switch is existed in the beginning/end of the line between fault j and load point i and 0 otherwise.

Functions:

C^{total}	Total cost function.
$SAIDI$	Total SAIDI function.
$C^{Inv/M}$	Total investment/ maintenance cost.
PW_t	Present worth factor.
C^{TCI}	Total customers interruption cost during the planning horizon.
$IC_z^{r_i}$	Customer's interruption cost of load point i during the interruption r_i.
u_i	Total interruption duration of load point i in the year.
λ_i	Total failure rate of load point i during all contingencies.
λ_{ij}	Failure rate of load point i during fault j.
r_{ij}	Interruption duration of load point i during fault j.
r_i	Average fault repair time should be tolerated by customers located at load point i for each contingency.

I. INTRODUCTION

HIGHER reliability of electric energy reliability is more demanded, followed by the proliferation of digital devices. Hereof, power distribution companies invest in their system reliability improvement [1]-[4]. However, fault occurrence in distribution systems is inevitable. Therefore, measures should be taken to reduce the adverse effect of fault events, to achieve higher reliability distribution systems [5].

One of the main operational functions to enhance reliability despite failures is the fault location, isolation, and service restoration (FLISR) process [2,3]. Once a fault occurs in a distribution system, all downstream customers are de-energized due to the circuit breaker trip. Afterward, the functions of the FLISR process are sequentially conducted to accelerate re-energizing as most as possible customers and repairing the faulted section. For each fault occurrence, there might be several possible restoration strategies depending on the location of available manual switches (MSs) and remote-controlled (automatic) switches (RCSs). However, all possible restoration strategies may not satisfy technical constraints, such as power flow constraints. Nevertheless, non-restorable customers will experience a service interruption equal to the repair time. Therefore, the improvement in system reliability highly depends on the location of switches and the switching time. Moreover, switching time in RCS is much less than MS. However, it is neither economical nor necessary to install switches in all possible locations. Therefore many studies are conducted in the literature to solve the optimal switch placement problem.

Billinton and Jonnavithula made one of the first attempts to optimally place switches in distribution systems in 1996 [1]. A non-linear algorithm is presented for the placement of RCSs in the distribution system [3]. In [6], a remote-controlled switch placement study is performed, regarding the annual load growth rate, where power flow constraints are not considered. A particle swarm optimization-based algorithm is developed in [7] and applied to sectionalizing and breaker placement problem, considering power flow constraints. Authors in [8] provide a new cost/benefit analysis for distribution network automation planning, considering earth fault levels. In [9], the paper extends the reliability evaluation procedure to incorporate the probability of RCS malfunctions. Furthermore, authors in [10,11], present a MIP formulation to integrate sectionalizing switches malfunction probability into sectionalizing switches placement problem. In [12], a mathematical model is introduced to optimally place fault indicators and remote-controlled switches. Authors in [13, 14], present models to consider the financial risk induced by the uncertainty in an RCS placement problem. In [15], the authors propose considering laterals as potential switch locations. Moreover, [16, 17,18] installed automatic switches for semi-self-financed deployment, in which the yearly profit of installing RCSs is considered in the financial process as a new investment. A MILP formulation is presented in [19] for the optimal placement of distribution automation equipment. A MILP is exploited in [20] to find the global optimum of the automated sectionalizing switch placement problem. In [21], the MILP model is extended to consider both short-circuit faults and earth faults. In [22], a new approach is offered to RCS allocation for enhancing the performance of restoration and optimizing reliability benefits with reasonable RCS cost.

Previous MILP formulation attempts considered linearization of the original MINLP problem, leading to approximate results. Moreover, while formulations scarcely consider power flow constraints, the ones considering it makes an approximate linearization on it. The major bottleneck in non-linear formulations relates to the product of two continuous terms and inverse terms, which are usually linearized by estimation/approximation methods with unsatisfactory accuracies.

The main contributions of this paper are listed below:

- Proposing a high-accuracy MILP Formulation for simultaneously finding the optimal location and number of automatic and manual switches with a predefined precision.
- Devising an innovative method for converting the product and the inverse of continues variables to mixed-integer linear relations, which lead to a high-accuracy MILP load flow formulation,
- Providing a decision support system for distribution system operators to investigate the trade-off between investing on the installment of automatic and manual switches and paying the penalty for load not served.

II. Problem Formulation

The purpose of the switch placement problem is to find the optimal layout of manual and automatic switches associated with the minimum value of the objective function; subject to the power flow constraints. A linear formulation is presented here for the allocation problem of the manual and automatic switches, as follows:

A. *Objective Function*

In this paper, a mixed-integer linear programming problem is formulated to minimize a cumulative economic and technical function as presented in (1)-(8).

$$\text{Min OF} = \psi \cdot \frac{C^{total}}{C^{min}} + (1-\psi) \cdot \frac{SAIDI}{SAIDI^{min}} \tag{1}$$

$$C^{total} = C^{Inv} + \sum_{t=1}^{n_t} PW_t \cdot (C^{M} + C^{TCI}) \tag{2}$$

$$SAIDI = \frac{\sum_{i=1}^{n_c} u_i \cdot N_i}{\sum_{i=1}^{n_c} N_i} \tag{3}$$

$$PW_t = \left(\frac{1}{NInt}\right)^t \tag{4}$$

$$NInt = (1+Inf)\cdot(1+Int) \tag{5}$$

$$C^{Inv} = N^{RCS}\cdot C^{Inv}_{RCS} + N^{MS}\cdot C^{Inv}_{MS} \tag{6}$$

$$C^{M} = N^{RCS}\cdot C^{M}_{RCS} + N^{MS}\cdot C^{M}_{MS} \tag{7}$$

$$C^{TCI} = \sum_{i=1}^{n_c} IC_z^{r_i} \tag{8}$$

The objective function (1) is the weighted summation of the normalized values for the total cost (C^{total}) and the system average interruption duration index ($SAIDI$). In addition, in (1), to equalize the effect of each index in the objective function, the indices are normalized with the optimal ones, due to distinct values of each index. The optimal value of each term of the objective function is calculated when another is not included in the OF. Moreover, the normalized values of indices are weighted using coefficients Ψ and (1- Ψ), which could be specified based on the distribution system operator (DSO) preference to invest on switching installment or pay the customer interruption penalty. In (1), C^{min} calculated when Ψ=1 and $SAIDI^{min}$ calculated when Ψ=0. The C^{total} is presented in (2), which includes three terms:

- The first term of (2) accounts for total installation costs of the deployed switches as presented in detailed in (6).
- The second part of (2) is total maintenance costs during the planning horizon that is represented in (7).
- The third term of (2) expresses total customer interruption costs during the planning horizon which is calculated using (8).

In order to show the time value of money, the nominal interest rate is evaluated through (4) and (5) and applied to (2), which considers the interest rate and the inflation rate for calculating the present value. Moreover, the SAIDI index is calculated through (3) considering all network faults.

B. *Reliability Calculations*

As in (9), u_i is yearly interruption duration in load point. The failure rate of load point i is calculated in (10) too. This value is a sum of all failure rate of contingencies since they are series elements in terms of reliability. In (11), the average fault repair time tolerated by customers located at load point i for each contingency is calculated [1].

$$u_i = \sum_{j=1}^{n_l} r_{ij} \cdot \lambda_{ij} \tag{9}$$

$$\lambda_i = \sum_{j=1}^{n_l} \lambda_{ij} \tag{10}$$

$$r_i = \frac{u_i}{\lambda_i} \tag{11}$$

Fig. 1 shows the flowchart for calculating the interruption duration of each load point. Here, impact of each contingency on each load point is investigated. Once the location and type of the fault is determined, the impacted load points by each contingency are listed. If a load point i is not affected by a contingency j, the interruption duration of load point i due to contingency j is zero. Otherwise, we check if there is a switch in the path of fault caused contingency j to load point i. If there is no switch in the path, the interruption duration of load point i in contingency j is r_j. Otherwise, a flag is raised to determine the type of the switch exists along the path of fault of contingency j to load point i. Then, another flag is raised if the load point i is supplied during contingency j. Once the value of all flags are determined, the formula

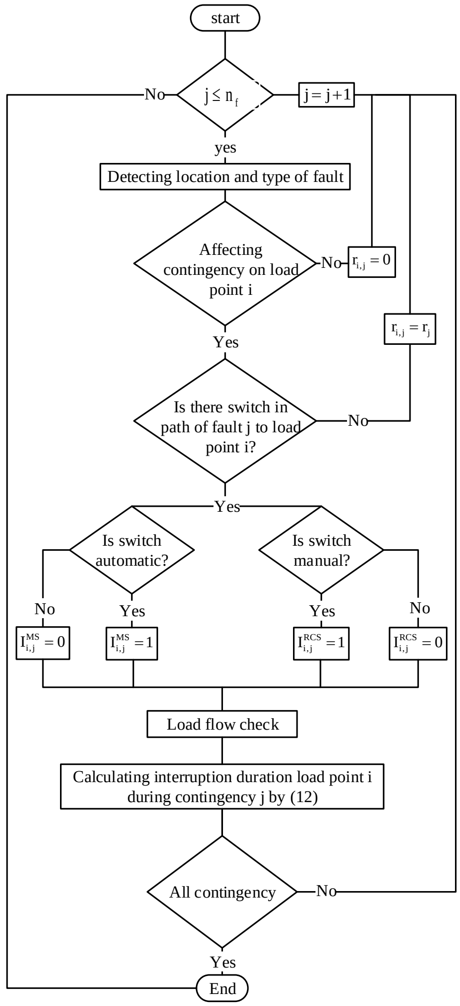

Fig. 1. Calculation the interruption duration of each load points.

given in (12) is employed to determine the interruption duration of load point i during contingency j.

Calculating interruption duration of each fault for load points is an important part of reliability problems. Interruption duration in this paper is calculated through (12)-(18). Interruption duration time in load point i due to fault j is calculated through (12), considering switches:

$$r_{ij} = (1-I_{ij}^{MS})\cdot(1-I_{ij}^{RCS})\cdot r_j + \left(\left(1-P^{RCS}\right)\cdot I_{ij}^{RCS}\cdot r_j + P^{RCS}\cdot I_{ij}^{RCS}\cdot t^{RCS}\right) + \left(\left(1-P^{MS}\right)\cdot I_{ij}^{MS}\cdot r_j + P^{MS}\cdot I_{ij}^{MS}\cdot t^{MS}\right) \quad (12)$$

In (12), $I_{i,j}^{MS}$ and $I_{i,j}^{RCS}$ are binary decision variables for manual and automatic switches respectively. Thus, if manual or automatic switch is installed in the path from fault j to load point i, the related binary variable is equal to 1 and 0 otherwise. The following rules applies:

- If I_n^{MS-B}/I_n^{RCS-B} equals to one, there should be a MS/RCS at the beginning of the upstream section of the fault.
- If $I_n^{MS-E}/\ I_n^{RCS-E}$ equals to one, there should be a MS/RCS at the end of the upstream section of the fault.

Equations (13) to (18) determine the state of the customers located at the j^th load point in case of occurrence of the i^th fault. They can be divided into two groups: equations (13) to (15) and equations (16) to (18), which relate to the existence of manual and automatic switches between the i^th fault and the j^th load point, respectively. These equations are written in a sequence, which meets the operational sequence of switches. As an instance, I_{ij}^{MS-B} in equation (13) is equal to one, only if there are no manual or automatic switches between fault location and load point. In other words, operation priority determines the switch. Thus, equation (13) recognizes whether there is any manual switch at the beginning of the n^th line or not. This equation includes three terms: first, recognition of existence of manual switch at the beginning of the n^th line; second, recognition of existence of manual switch between fault location and the location of the switch located before the n^th switch; third, recognition of existence of automatic switch between fault location and the location of the switch located before the n^th switch. In this manner, if there is a manual switch between these locations, the second term of the equation equals to zero. With the same procedure for an automatic switch, the third term of the equation will be equal to zero. Equation (14) is similar to equation (13), but it recognizes the existence of a manual switch at the end of the line. Equation (15) is exclusive or (*XOR*) of I_{ij}^{MS-B} and I_{ij}^{MS-E}, which finally recognizes the existence of a manual switch between fault location and load point. Similarly, equations (16) to (18) detect the presence of automatic switches.

In (13)-(18), k denotes the candidate location of switches and k_1 and k_2 denote the beginning and the end of the lines from location of fault to location of past k^{th} switch.

$$I_{ij}^{MS-B} = \sum_{n=1}^{s} I_n^{MS-B}\cdot\prod_{k=1}^{n-1}\left(\left(1-I_k^{MS-B}\right)\cdot\left(1-I_k^{MS-E}\right)\right) \times\prod_{k=1}^{n}\left(1-I_k^{RCS-E}\right)\cdot\prod_{k=1}^{n}\left(1-I_k^{RCS-B}\right) \quad (13)$$

$$I_{ij}^{MS-E} = \sum_{n=1}^{s} I_n^{MS-E}\cdot\prod_{k=1}^{n-1}\left(\left(1-I_k^{MS-B}\right)\cdot\left(1-I_k^{MS-E}\right)\right) \times\prod_{k=1}^{n}\left(1-I_k^{RCS-E}\right)\cdot\prod_{k=1}^{n}\left(1-I_k^{RCS-B}\right) \quad (14)$$

$$I_{ij}^{MS} = I_{ij}^{MS-E} + I_{ij}^{MS-B} - I_{ij}^{MS-E}.I_{ij}^{MS-B} \quad (15)$$

$$I_{ij}^{RCS-B} = \sum_{n=1}^{s} I_n^{RCS-B}\cdot\prod_{k=1}^{n-1}\left((1-I_k^{RCS-B})\cdot(1-I_k^{RCS-E})\right) \times\prod_{k=1}^{n}\left(1-I_k^{MS-E}\right)\cdot\prod_{k=1}^{n}\left(1-I_k^{MS-B}\right) \quad (16)$$

$$I_{ij}^{RCS-E} = \sum_{n=1}^{s} I_n^{RCS-E}\cdot\prod_{k=1}^{n-1}\left((1-I_k^{RCS-B})\cdot(1-I_k^{RCS-E})\right) \times\prod_{k=1}^{n}\left(1-I_k^{MS-E}\right)\cdot\prod_{k=1}^{n}\left(1-I_k^{MS-B}\right) \quad (17)$$

$$I_{ij}^{RCS} = I_{ij}^{RCS-E} + I_{ij}^{RCS-B} - I_{ij}^{RCS-E}.I_{ij}^{RCS-B} \quad (18)$$

C. *Constraints*

Power flow is the underlining problem for power system analysis [23]. In this paper, power flow constraints are considered to determine the feasibility of possible restoration strategies. To such aim, a linear power flow method is presented to investigate the following constraints:

1) *Power Flow*

The proposed method in [24], proves the accuracy of the convex relaxation constructing a group of monotonic series with constraints that ensures that the optimal solution of the second-order cone program can be converted to an optimal solution of the original AC optimal power flow. Authors in [25] suggested a service restoration method, which uses active management of the distribution network considering the coordinated control of the available switches, distributed generation units, and the operation of on-load tap changers. This approach minimizes the out of service area considering customer priorities and the number of switch operations.

The linearized power flow problem formulation is presented in (19)-(22), while its mathematical basis is discussed in more details in the solution method section. The power flow in (19) can be extended to (21). Since $1/V_i^*$ in (21) is nonlinear, it can be written as (22), using the equations expressed in the solution method section, where V_i is the voltage on node i and IV_i^* is the inverse of conjugate voltage on node i.

$$S_i = V_i \cdot I_{iw}^* \tag{19}$$

$$S_i^* = V_i^* \cdot I_{iw} \tag{20}$$

$$\frac{1}{V_i^*}.S_i^* = Y_{ii} \cdot V_i + \sum_{w=1, w\neq i}^{n} Y_{iw} \cdot V_w \tag{21}$$

$$IV_i^*.S_i^* = Y_{ii} \cdot V_i + \sum_{w=1, w\neq i}^{n} Y_{iw} \cdot V_w \tag{22}$$

2) *Budget*

Total investment cost of the switch placement problem should not exceed the available budget that is constrained as (23).

$$C^{Inv} \le Budget \tag{23}$$

3) *Voltage constraint*

All customers should be supplied within the permissible voltage margin during restoration process.

$$V^{min} \le V_i \le V^{max} \tag{24}$$

4) *Thermal Capacity Constraint*

Restoration strategies should satisfy the thermal constraint on the feeder current.

$$|I_{iw}| \le I_{iw-thermal} \tag{25}$$

5) *Network Radiality*

The distribution system operation is radially operated; therefore, the restorable customers should only be fed from one supply point. Accordingly, as an example in RBTS4 switches:

$$\sum\left(I_n^{MS-B} + I_n^{MS-E} + I_n^{RCS-B} + I_n^{RCS-E} + I_n^{RCT}\right) \ge 1$$
$$\forall n \in \left\{feeder\ h, p \,\middle|\, (h,p) \subseteq \{(f1, f7), (f2, f6), (f2, f5), (f3, f4), (f5, f6)\}\right\} \tag{26}$$

III. Linear Conversion Formulation

In most studies in power systems, research objectives require accurate modeling of the network constraints. Using nonlinear models of the system might causes non-uniqueness of the procured solutions or failure to find an optimal solution. Thus, the procured solutions based on nonlinear models are not necessarily the best one for planning and operation of the power systems. This calls for developing a linearized approach without approximation error. The presented solution method attempted to address this gap to a reasonable extent.

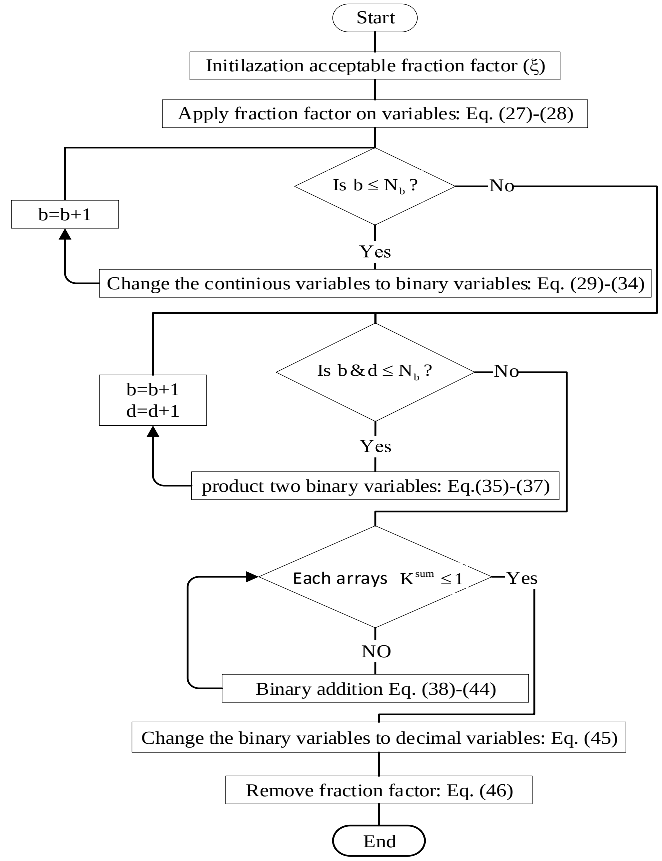

Fig. 2. Product of two continuous variables linear conversion.

A. *Product of Two Continuous Variables Linear Conversion:*

One major challenge is to linearize the product of two continuous variables [24]. An algorithm is introduced here to convert the product of two continuous variables into a mixed-integer linear formula without making an approximation. The flowchart of the developed algorithm is shown in Fig. 2.

The first step is to convert the considered continuous terms into an equivalent integer number. The method presented here can multiply decimal numbers by any precision. Here, the continuous terms V_1 , V_2 are converted to integer variables of $X_1(0)$, $X_2(0)$ using (27) and (28), where the precision is determined based on the choice of the coefficient ξ. Thus, this coefficient is determined according to the expected accuracy of the calculation.

Moreover, the approximation in the equations can be controlled by a coefficient, which can be adjusted by the user. This factor determines the calculation limits (the number of significant digits). For example, setting 4 significant digit results in a switch placement solution with load flow and with the answer, which enjoys 4 significant digits accuracy (no approximation is included in the calculations).

The following step is to convert the procured integer variables into the binary domain given that the basis of this method is computing in the binary domain. Thus, using (29) to (31), X_1 is stored in the binary form in Y_1. N_b and N_{con} are the maximum number of binary digits required to represent X_1 and N_b as binary, respectively. N_{sum} is equal to $2N_b - 1$. For example, Table I illustrated the product of two continuous variables using linear conversion.

$$X_1(0) = \zeta \cdot V_1 \tag{27}$$

$$X_2(0) = \zeta \cdot V_2 \tag{28}$$

$$X_1(b+1) \le \frac{X_1(b)}{2} \quad \forall b \in N_b \tag{29}$$

$$X_1(b+1) \ge \frac{X_1(b) - 1}{2} \quad \forall b \in N_b \tag{30}$$

$$Y_1(b+1) = X_1(b) - 2 \cdot X_1(b+1) \quad \forall b \in N_b \tag{31}$$

$$X_2(b+1) \le \frac{X_2(b)}{2} \quad \forall b \in N_b \tag{32}$$

$$X_2(b+1) \ge \frac{X_2(b) - 1}{2} \quad \forall b \in N_b \tag{33}$$

$$Y_2(b+1) = X_2(b) - 2 \cdot X_2(b+1) \quad \forall b \in N_b \tag{34}$$

Now that two binary variables are available as Y_1 , Y_2, the linear product of two binary variables is performed using (35) to (37) [27]. A variable with strings 0 or 1 and dimensions (b, d) is created in matrix Z.

$$Z(b,d) \le Y_1(b) \quad \forall b,d \in N_b \tag{35}$$

$$Z(b,d) \le Y_2(d) \quad \forall b,d \in N_b \tag{36}$$

$$Z(b,d) \ge Y_1(b) + Y_2(d) - 1 \quad \forall b,d \in N_b \tag{37}$$

All of the rows in each column of Z must be added as a binary variable. The value of 0 or 1 which is obtained from the sum of the rows of the previous column, must be added to the total sum of the rows of the current column.

Finally, the variable K as the product of the two variables X_1, X_2 with 0 and 1 arrays, could be obtained. The aforementioned steps are shown in equations (38) to (44).

$$K^{shift}_{(b,d+b-1)} = Z(b,d) \quad \forall b,d \in N_b \tag{38}$$

$$K^{sum}_{(m)} = \sum_{\forall b \in N_b} (K^{shift}_{(b,m)}) \quad \forall m \in N_{sum} \tag{39}$$

$$K^{con}_{(m,0)} = K^{sum}_{(m)} \quad \forall m \in N_{sum} \tag{40}$$

$$K^{con}_{(h,f)} \le \frac{K^{con}_{(h-1,f)}}{2} \quad \forall h \in N_{sum}, f \in N_{con} \tag{41}$$

$$K^{con}_{(h,f)} \ge \frac{K^{con}_{(h,f)} - 1}{2} \quad \forall h \in N_{sum}, f \in N_{con} \tag{42}$$

$$K^{binary}_{(h,f)} = K^{con}_{(h-1,f)} - 2 \cdot K^{con}_{(h,f)} \quad \forall h \in N_{sum}, f \in N_{con} \tag{43}$$

$$K(h) = K^{binary}_{(h,N_{con})} + \sum_{e} \sum_{\forall p<h} (K^{binary}_{(p,N_{con}-e)}) \quad \forall h,p \in N_{sum} \text{ and } e \in [1, N_{con}] \tag{44}$$

To achieve the decimal value of the product of the two variables X_1, X_2, it is necessary to transform the binary value to the corresponding decimal, which can be calculated based on the Eq. (45), and the value of the variable S can be obtained.

$$S = \sum_{\forall b \in 3 \times N_b} \left(K(b) \cdot 2^{(b-1)} \right) \tag{45}$$

To obtain the product of two variables V_1, V_2, and in order to eliminate the effect of the integer coefficient, variable S must be divided to ξ^2 as shown in Eq. (46).

$$M^{(v_1 \cdot v_2)} = \frac{S}{\zeta^2} \tag{46}$$

B. Inverse of a Continuous Variable Linear Conversion:

The inverse of the continuous variable V_i in the power flow equation (21) needs to be converted to a linear form as in (22). To generalize, the inverse of a continuous variable x is considered here. In this section, an innovative method is introduced for the linearization of the non-linear term $1/x$ using equations (52) through (53). In this method, the new continuous variable y replaces the non-linear term $1/x$ which leads to:

$$x.y = 1 \tag{47}$$

Considering complex nature for x and y, equation (47) requires the following for magnitude and angle:

$$x = r_x \angle \varphi_x \tag{48}$$

$$y = r_y \angle \varphi_y \tag{49}$$

$$r_x . r_y = 1 \tag{50}$$

$$\angle \varphi_x = -\angle \varphi_y \tag{51}$$

The equation for magnitude in (50) is established using equation (52). In this regard, the linear form of product of two continuous variables in the form of equation (46) is as follows:

$$M^{(M^{(Re_x \cdot Re_x)} \cdot M^{(Re_y \cdot Re_y)})} + M^{(M^{(Re_x \cdot Re_x)} \cdot M^{(Im_y \cdot Im_y)})} + \ldots$$
$$M^{(M^{(Im_x \cdot Im_x)} \cdot M^{(Re_y \cdot Re_y)})} + M^{(M^{(Im_x \cdot Im_x)} \cdot M^{(Im_y \cdot Im_y)})} = 1 \tag{52}$$

The equation for angle in (51) is established using equation (53). In this regard, the linear form of product of two continuous variables in the form of equation (46) is as follows:

$$M^{(Im_x \cdot Re_y)} = M^{(Im_y \cdot Re_x)} \tag{53}$$

TABLE I. ILLUSTRATING PRODUCT OF TWO CONTINUOUS VARIABLES LINEAR CONVERSION.

V_1	V_2	ξ	N_b	N_{sum}	N_{con}	$X_1(0)$	$X_2(0)$	Y_1=1100101
10.1	9.7	10	7	13	3	101	97	Y_2=1100001

Z	K^{shift}	K^{sum} =1111013110011			
		K^{binary}			K
1100001	0000001100001				
0000000	0000011000010				
1100001	0000000000000	1- 001	6- 001	11- 001	10011001000101
0000000	0000000000000	2- 001	7- 011	12- 001	S=9797
0000000	0011000010000	3- 000	8- 001	13- 001	
1100001	0000000000000	4- 000	9- 000		$M^{(10.1*9.7)}$ =97.97
1100001	1100001000000	5- 001	10-001		

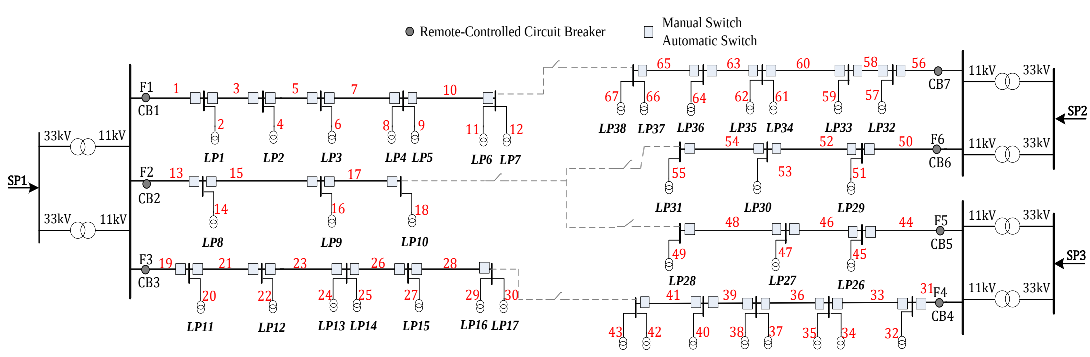

Fig. 3. Single line diagram of RBTS4 [26].

IV. Case Study and Results

The proposed method is applied to bus number four of Roy Billinton Test System (RBTS4), depicted in Fig. 3 [28]. The failure rates of network equipment are derived from [28]. It is assumed that various faults possess the same occurrence probabilities and different occurrence rates. In addition, the single failure is considered in the paper, while consideration of multiple failures is let for future work. The automatic switching time and manual switching times are assumed equal to 30 seconds and one hour, respectively. The installation costs of the automatic and manual switches are set equal to $15,000 and $6,000 respectively, while the planning horizon is considered 15 years. Moreover, the interruption cost functions for different types of customers are adopted from [3] and the probability of automatic and manual switching are 98.5% and 95%.

The performance of the proposed method is numerically studied in three cases. In the first case, manual switches are optimally allocated in the distribution system. The second case determines the optimal location and number of automatic switches. In addition, both manual and automatic switches are allocated in case 3. The optimal switch placements in all cases are shown in Table II and the measures for all cases are shown in Table III.

Table II. Optimal Switch Placement.

($Budget = 150$ (US 1000\$), $\Psi = 0.6$)

Test Case	Normally closed switch placement: Manual	Normally closed switch placement: Remote Control	Number of switches
Case1	3D,5D,7U,10D,15D, 17D,21D,23U,26D, 28U,33D,36D,36U, 39D,41U,46U,48U, 52D,54D,58D,60D, 65D	---	22
Case2	---	7D,10D,17D,23U,26D,36D, 39D,46U,54D,63D	10
Case3	15D,17D,36U,46U,48U, 54D,60D	7D,10D,23U,26D,28U,39D, 63D	14

Table III. Overall Reliability Indices for Original And Different Equipped System.

Test case	Obj. Fun (pu)	C^{Total} (US $1k)	C^{INV} (US $1k)	C^{M} US $1k)	SAIDI (h/year.cust)	ASAI (pu)	ENS (kW/ year)
Base	1.006	1713.4	0	0	2.2717	0.999740	43829
Case1	0.649	1013.6	132	15.02	1.3544	0.999845	26183
Case2	0.585	897.43	150	17.07	1.1837	0.999864	23282
Case3	0.581	898.64	147	16.73	1.1632	0.999867	23495

According to Table II, in case 1, 22 manual switches are allocated; as a result in Table III, the distribution automation cost is $132k and C^{Total} is equal to $1013.6k. At the same time, SAIDI decreases from 2.2717 to 1.3544 [h/year.cust]. In case 2, 10 automatic switches are allocated in the distribution system. The investment cost is $150k and C^{Total} in this case is equal to $897.43k. In addition, SAIDI decreases to 1.1837 [h/year.cust]. Moreover, as shown in Table II, two RCSs installed at 5D and 7U of feeder 1 which "U" and "D" indicate upstream and downstream of a line section, respectively.

Proper expansion of the automatic switches in the distribution system assures reliability improvement. However, it is conceivable that the overall budget might run over. Economically, the installation costs of the manual switches are less than the automatic ones, although the adequate reliability level may not be reached. Thus, both manual and automatic switches are allocated in the distribution system by solving the problem in case 3. As shown in Table II and III, 7 manual switches and 7 automatic switches are selected. Also, in this case, the investment cost is $147k and SAIDI is 1.1632 [h/year.cust].

The annual interruption cost of each load point regarding the three discussed cases and the base case (the existing system) is shown in Fig. 4. As it is shown in Fig. 4, CIC for most buses in the last case decreases in comparison to two other cases, while the CIC for some of the buses significantly increases. However, this increase is much less than that of the decrease in other buses.

V. Discussions

Although the effectiveness of the proposed method is presented in Section IV, several parameters such as objective function weighting coefficients as well as total planning investment might affect the optimal manual and automatic switches locations and numbers. Accordingly, sensitivity analysis is studied to evaluate the consequences of these parameters on the performance of the proposed approach.

A. *Effectiveness of the Proposed MILP Formulation*

In this section, the obtained results of the MILP formulation in this paper, the MINLP formulation in [3] and the MILP formulation in [10] are compared to show the effectiveness of the proposed MILP formulation. The investment budget is increased to $400k as in [3,10]. Moreover, Ψ is set equal to 1 to convert the objective function into the cost objective as in references [3,10]. The results once utilizing the three formulations are shown in Table IV. Four manual and 12 automatic switches are installed using the MILP formulation of this paper, while 14 (13) manual and 10 (10) automatic switches are installed using the MINLP (MILP) formulation of [3] ([10]). Furthermore, the total investment cost of our MILP formulation is $203k that is 12.8% (10.5%) lower than that of the MINLP (MILP) formulation of [3] ([10]), while SAIDI is 7.8% (2.9%) decreased. Thus, the MILP formulation prescribed fewer switches with lower achieved SAIDI, compared to that of the MINLP (MILP) formulation in [3] ([10]).

Table IV. Comparison The Results in This Paper And [3,10].

$Budget = 400$ (US $1k)

Test Case	Normally closed switch placement: Manual	Normally closed switch placement: Automatic	C^{Total} (US $1k)	SAIDI (h/year .cust)	CIC^{Tot} (US $1k)
Linear model of this paper	15D,52D,58D,63U	7D,10D,17D,23U,26D,28U,36D,39U,46U, 48U,54D,65D	1026	1.1419	799
Ref [3]	5U,10D,15D,21U,23U,28D,33U,39D,46U, 48U,50U,54D, 54U, 63D	7U,15U,17D,26D,36D,41D,48D,52U,58D, 63U	1185	1.2391	924
Ref [10]	5U,13U,15D,17U,23U,36U,44U,46D,48U, 50U,52D,54D,60U	10U,15U,17D,28U, 41U,46U,48D,52U, 54U,65U	1084	1.1751	831

B. *Sensitivity Study on the Weighting Coefficient*

As noted in problem formulation, both SAIDI and C^{Total} are considered in the proposed objective function. The priority of SAIDI and C^{Total} are defined by weighting coefficients. The weight coefficient of C^{Total} and the weight coefficient of SAIDI are Ψ and (1- Ψ), respectively. To evaluate the impact of changes in SAIDI as customer welfare and C^{Total} as a total cost of the system, sensitivity analysis is performed on the several weighting

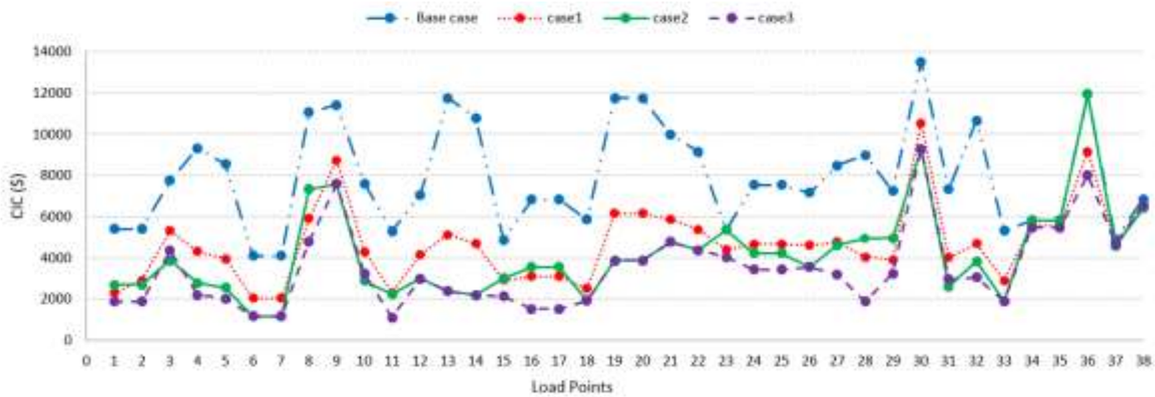

Fig. 4. Annual interruption cost of load points.

coefficients. To this end, the proposed formulation for case 3 is tested for six different cases and the results are shown in Table V. As reported of this table, when minimizing the SAIDI index assumed as the only objective of the problem, total cost and CIC increases. In this case, only automatic switches are selected to be installed in the distribution system. However, load points with the greater value of interruption cost possess higher priority, when the C^{Total} index is assumed as the only objective function, and SAIDI increases. C^{Total} and SAIDI for all cases in Table V are shown in Fig. 5. As shown in this figure, the total cost of the system is decreased when the weighting coefficient is increased from 0 to 1. Moreover, SAIDI increases in this state. It is up to the preference of the distribution system operator to decide on the appropriate weight coefficient.

TABLE V. SENSITIVITY ANALYSIS ON WEIGHTING COEFFICIENTS ON CASE 3.

Ψ (pu)	C^{Total} (US $1k)	C^{INV} (US $1k)	SAIDI (h/year cust)	ASAI (pu)	ENS (kW/ year)	Number of switches	
						RCS	*MS*
0	1047.17	150	1.0734	0.999877	26560	10	0
0.2	929.30	150	1.1091	0.999873	23967	8	5
0.4	910.72	147	1.1407	0.999869	23710	9	2
0.6	898.64	147	1.1632	0.999867	23495	7	7
0.8	888.09	150	1.1579	0.999866	23166	8	5
1	884.94	150	1.1772	0.999865	23256	8	5

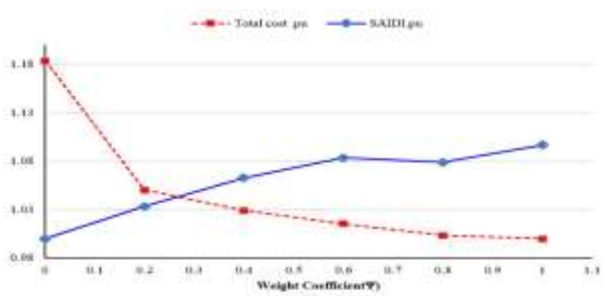

Fig. 5. Total cost and SAIDI for all cases.

C. Sensitivity Analysis on Budget

The relation between the optimum number and locations of switches, and the total planning investment cost (budget) is studied. The proportion of the rated planning investment to its related cost-benefit is one of the most important issues in the planning of the distribution network. The solution to the presented problem highly depends on economic parameters like rate of return, interest, and inflation rates. Moreover, in this case, the optimal layout of joint automatic and manual switches without considering the economic constraints is deduced and details of the corresponded layout are reported. The effect of planning investment on the proposed approach is conducted in this section and the obtained results are shown in Table VI. In addition, these analyses are performed in case 3. As shown in Table VI, the total cost decreases if the spent planning investment increases from $30k to about $240k. However, the total cost will increase if the planning investment increases from $240k to $300k.

Moreover, when the planning investment is increased, the usage of automatic switches is increased and while it is decreased for the manual switches. SAIDI is decreased if the planning investment is increased. Fig. 6 shows that the number of automatic switches is increased by incrementing outlay on automating the study distribution system; on the other hand, the total number of manual switches is decreased. Furthermore, the increase in the budget shows a saturation effect on reliability improvement. Here, while the increase in the budget from $30k to $180k results in a 32% improvement in SAIDI, further increasing the budget from $180k to $300k only results in 8.1% of improvement in SAIDI as the reliability measure.

TABLE VI. SENSITIVITY ANALYSIS ON BUDGET.

Budget (US $1k)	C^{Total} (US $1k)	C^{INV} (US $1k)	SAIDI (h/yr cust)	ASAI (pu)	ENS (kW/ year)	Number of switches	
						RCS	*MC*
30	1333.15	30	1.6479	0.999811	33884	0	5
60	1138.01	60	1.4155	0.999838	29215	2	5
90	1018.53	90	1.2944	0.999852	26464	4	5
120	955.15	120	1.1775	0.999865	24792	6	5
150	898.64	147	1.1632	0.999867	23495	7	7
180	844.48	177	1.1211	0.999872	22102	11	2
210	819.26	198	1.0843	0.999876	21463	12	3
240	777.60	228	1.0578	0.999879	20331	14	3
270	757.91	258	1.0305	0.999882	19799	16	3
300	757.91	258	1.0305	0.999882	19799	16	3

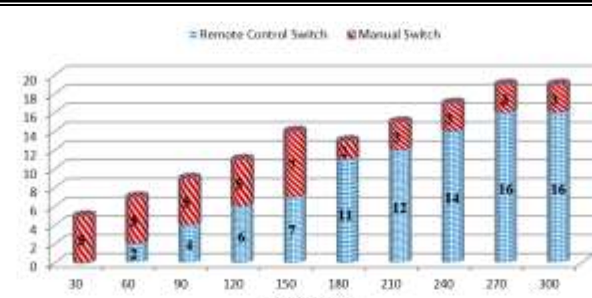

Fig. 6. Number of switches base on budget.

D. Sensitivity Analysis on the Length of Planning Horizon

The sensitivity of the length of the planning horizon is analyzed, in two different scenarios, one with a budget of $150k and the other with an unlimited budget. Table VII shows the results of these two scenarios with different planning periods. The shorter planning horizon, the investment cost is more significant due to the lower CIC. Therefore, less switch is located, in both scenarios, yet less effect of the limited budget is observed.

TABLE VII. SENSITIVITY ANALYSIS ON PLANNING HORIZON.

Budget (US $1k)	Planning Horizon	C^{Total} (US $1k)	C^{INV} (US $1k)	Number of switches	
				RCS	*MS*
150	5	485.24	123	5	8
	10	632.53	132	6	7
	15	898.64	147	7	7
	20	1164.82	147	7	7
inf	5	410.35	159	9	4
	10	554.14	210	12	5
	15	757.91	258	16	3
	20	941.32	327	21	2

Fig. 7 shows the share of each switch type located in each scenario. Only a limited number of switches are located in each planning horizon, even when the budget is infinite. It is shown that increasing the planning horizon, increases the share of RCSs since the total CIC is increased, thus allocating a larger number of RCSs is justifiable.

E. *The Role of the Proposed Power Flow Formulation*

In this section, the role of the proposed power flow formulation in the switch placement problem and the runtime of the program is discussed. According to the results presented in Table VIII, where the power flow formulation is not carried out, assuming all loads to be supplied by adjacent feeders, SAIDI and CIC decrease. This reduction is due to less interruption time in the load points, which cannot be supplied if the power flow is considered since the thermal limitation of feeders does not allow using the adjacent feeder. Running the problem with the proposed power flow formulation increases program runtime about 35 seconds, which is a 44% increase in computation time. However, this runtime increment is acceptable in planning studies.

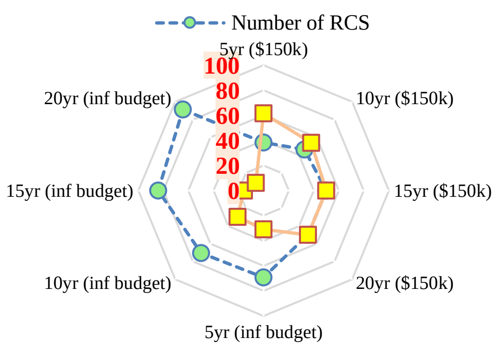

Fig. 7. Percentage of switches located for budget and planning horizon.

TABLE VIII. COMPARISON THE RESULTS WITH AND WITHOUT POWER FLOW.

$Budget = 150$ (US 1000$), $\psi = 0.6$,						
Test case	Number of switches		Runtime (sec)	C^{Total} (US $1k)	SAIDI (h/year.cust)	C^{INV} (US $1k)
	RCS	*MS*				
With PF	7	7	114	898.64	1.1632	147
Without PF	6	9	79	642.83	0.8162	144

F. *Sensitivity Analysis on Coefficient ξ*

The linear conversion formulation is presented in Section III. According to equations (27) and (28), coefficient ξ determines the accuracy of the calculations following the number of considered significant digits. In this section, the program runtime for different values of ξ is discussed. Table IX shows the results of this analysis under the condition that six significant digits are considered in the optimization program. Where ξ is varied from 1 to 6, the program runtime and optimization results are presented. Obviously, by the increment of ξ, the optimization program takes a longer time to finish. At ξ=1, optimization takes 94 seconds, while for ξ=6 it takes 163 seconds. On the other hand, the optimization result shows a 0.01% improvement. It can be inferred that by increasing ξ not only program runtime but also runtime increment rate increases. However, the procured results of the optimization function do not enjoy a significant improvement. Therefore, it can be concluded that an average value for this parameter should be considered for higher efficiency and better results, i.e. ξ=4.

TABLE IX. SENSITIVITY ANALYSIS ON COEFFICIENT ξ.

ξ	1	2	3	4	5	6
Runtime (sec)	94	99	106	114	136	163
C^{Total}(US $1k)	898.732	898.691	898.656	898.643	898.639	898.637

G. *Implementing to A Real and Large Distribution Network*

To explore the practicality and scalability of the proposed method, the model is applied to a real distribution network in Finland. The single line diagram of the network is illustrated in Fig. 8, courtesy of [29]. The network comprises six feeders. Each feeder is connected to another feeder by a tie switch. The same assumptions as for the test network, are considered in this network. Simulation results are brought in Table X. Decreases in total cost as 73%, 79% and 82% are achieved for the first three cases, respectively. Furthermore, SAIDI is decreased from 25.8 to 4.01. The program runtime is 403 seconds, which shows the functionality of the proposed method.

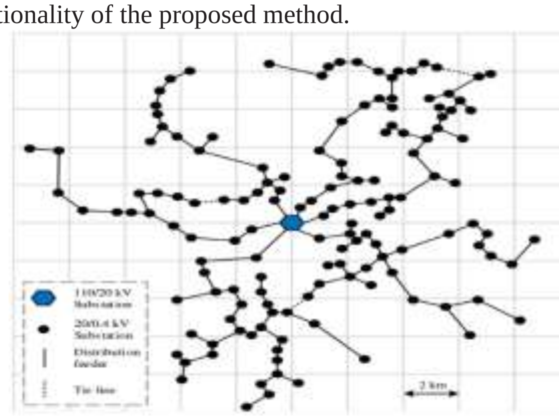

Fig. 8. Single line diagram of Finnish distribution network [29].

TABLE X. SYSTEM COSTS AND RELIABILITY INDICES IN THE FINNISH DISTRIBUTION NETWORK.

Test case	Obj. Fun (pu)	C^{Total} (US $1k)	C^{INV} (US $1k)	C^{RCS} (No.)	C^{MS} (No.)	SAIDI (h/year.cust)
Base	1.106	5450.2	0	0	0	25.814
Case1	0.501	1425.7	150	0	25	5.0016
Case2	0.424	1123.5	150	10	0	4.6512
Case3	0.401	982.3	150	6	10	4.0152

VI. CONCLUSIONS

In this paper, a problem reformulation is introduced to optimally deploy manual and automatic switches in the distribution system to improve system reliability. According to the proposed approach, the reliability level of the distribution system is enhanced, while minimizing the total cost. The objective is to minimize SAIDI, CIC, and investment as well as maintenance costs.

An innovative high-accuracy linear conversion is presented for the product and the inverse of continuous variables. Thus, a new MILP power flow solution is demonstrated without any approximation error too.

The results reveal that when simultaneous automatic and manual switches are implemented, both SAIDI and total cost are decreased compared to the optimal allocation of only a single type of switches.

In addition, sensitivity analysis revealed that the result of the planning is sensitive to the management strategy demonstrated as the weighting coefficients. Another sensitivity study showed that incrementing the outlay on switch development leads to an increase in the share of automatic switches versus the manual ones. The length of the planning horizon is also studied as it projects a contrast between CIC and budget: the investment cost is significant, due to the lower CIC while the length planning horizon is short.

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