Strategic Convergence Bidding in Nodal Electricity Markets: Optimal Bid Selection and Market Implications

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***Abstract*—Convergence bidding (CB), a.k.a., virtual bidding (VB), is a market mechanism facilitated by Independent System Operators (ISOs) in wholesale electricity markets to help lower the *gap* between the prices in the day-ahead market (DAM) and those in the real-time market (RTM). In this paper, we seek to answer two questions: 1) how can a strategic market participant maximize its profit when submitting CBs? 2) how can such strategically placed CBs affect the price gaps? Answering these questions is not straightforward but the results are insightful. The bidding problem in this context is a bi-level optimization, where the upper-level is about maximizing the profit for the convergence bidder and the lower-level is the economic dispatch problem. By solving the formulated bidding problem, we investigate the impact of strategic CBs on the DAM and RTM locational marginal prices (LMPs) under various practical scenarios. We demonstrate the scenarios under which a strategic CB, whether on its own, or when it is submitted jointly with a physical demand or physical supply bid, can or cannot work as intended, and result in decreasing or increasing the price gap in nodal electricity markets. We also examine how the performance of strategic CBs can be affected by uncertainty in demand or generation bids as well as physical contingencies in the power system, such as transmission line tripping. Special cases, such as net-zero convergence bidding are studied. The long-term performance of strategic convergence bidding is investigated. The above and other market implications are discussed in multiple case studies.**

***Keywords*: Convergence bids, virtual bids, strategic bidding, nodal prices, price gap, market efficiency, price divergence.**

NOMENCLATURE

$\mathcal{N}$	set of all buses
$\mathcal{L}$	set of all transmission lines
t	Index of hour-based time slot
d	Superscript for the day ahead market
r	Superscript for the real time market
g	Superscript for the generation resources
l	Superscript for the demands
b	Superscript for the convergence bids
$+/-$	Superscript for increased/decreased generation
n, m	Subscript indicating the nodes
k	Subscript indicating the scenarios
j	Subscript indicating the resources
H	Susceptance of the network lines
$\overline{C}$	Capacities of transmission lines
Δ	Difference between actual and cleared demand
M	A sufficiently large number
c	Cost of convergence bid
a	Price bid component
P	Cleared MWh in the market
θ	Voltage angle of the buses
ρ	The probability of scenarios
λ	Market cleating price
μ	Dual variables corresponding to the inequality constraints of market participants
β	Dual variables corresponding to the inequality constraints of transmission network lines
$\underline{\cdot}\,/\,\overline{\cdot}$	Indicates the minimum/maximum bound
X	A vector containing the left hand side of all inequality constraints in (1c)-(1f) or (2c)-(2e)
Y	A vector containing the dual variables of all inequality constraints in (1c)-(1f) or (2c)-(2e)
z	A vector containing the binary variables

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I. INTRODUCTION

A. Background and Motivation

The term convergence bid (CB) is used by the California ISO to refer to a class of financial bids that can be submitted to the ISO's day-ahead market (DAM), without involving physical power generation or power consumption [1]. Other ISOs also use similar bidding concepts that are sometimes referred to as virtual Bid (VB), such as in the PJM Interconnection [2]. These financial/virtual supply and demand bids settle first at the DAM and accordingly charged or credited to the market participants based on the locational marginal prices (LMPs) in the DAM; and subsequently liquidated with the opposite sell/buy position based on the LMPs in the RTM.

From the viewpoint of ISOs, CBs are introduced to achieve *price convergence*, i.e., to reduce the *price gap* between DAM and RTM. Price convergence across DAM and RTM is an important indicator of a robust and optimal electricity market [3]–[5]. As explained in [6], price convergence improves the efficiency of the day-ahead commitment and energy schedules, reduces the cost of hedging, allows for efficient settlement of Financial Transmission Right (FTRs), and makes it advantageous for parties to utilize the liquidity provided in the market. ISOs expect convergence bidders to provide highly competitive pressure to arbitrage between the markets and achieve price convergence. Therefore, in an ideal electricity market, there should be little or no gap between the prices in the DAM and the prices in the RTM. CBs may also help by increasing the competition in electricity markets which traditionally have problem with the low number of market participants [7].

Any financial entity that meets the financial credit requirements of the ISO is eligible to submit CBs. This provides a wide range of investors with new and major opportunities to participant in electricity markets without owning or operating any physical asset. Of course, the owners and operators of physical assets too may submit CBs in any bus - and not just the buses where their assets are located - to hedge against volatility in RTM LMPs or for speculative purposes.

Thus, in this paper, we seek to answer a series of questions related to CBs in nodal markets: **(1)** How can a convergence bidder strategically submit its bids to maximize its own profit? **(2)** What if such convergence bidder also owns physical assets that submit physical demand and supply bids? **(3)** What is the impact of strategic CBs on the market? **(4)** How can a CB at a bus or a group of coordinated CBs at multiple buses, affect LMPs at difference buses, in terms of decreasing (desired) or increasing (not desired) the price gap between the DAM and RTM prices? **(5)** What is the impact of uncertainty?

B. Literature Review

The literature on convergence bidding can be broadly divided into four groups. First, there are studies that focus on evaluating how CBs may have affected the existing ISO markets since they started adopting CBs. For example, a statistical analysis on how CBs have affected the prices in the CAISO market is presented in [8]. The effect of CBs in presence of fast ramping constraints in the current CAISO market is also studied in [4]. A critical analysis of the CB incidents that are adopted by PJM is also done in [9].

Second, there are studies that discuss how convergence bidding has the potential to correct the wrong schedules in deterministic markets. For example, the potential for major inefficiencies of deterministic day-ahead scheduling is discussed in [10], [11]. It is showed that convergence bidding, together with some self-scheduling by large slow-start conventional generators, may overcome these inefficiencies. In another related study in [12], the authors show that deterministic system operation with convergence bidding approximates the results of stochastic system operation, under certain assumptions, helping to correct the wrong deterministic schedules.

Third, there are studies that focus on the fundamental characteristics of CBs to analytically investigate the impact of CBs on the market. For example, in [13], [14] the authors studied how the network topology and transmission line congestion can influence the impact of CBs in nodal electricity markets. In [15]–[17], the authors examine how CBs may enhance market efficiency or help prevent price spikes or market power. Analytical studies of CBs also include the work in [18]–[21], where the authors showed that convergence bidders can manipulate the Financial Transmission Rights (FTR) auctions in electricity markets for their own benefit. Furthermore, in [22], the authors have developed an analytical model to characterize convergence bidding under imperfect market competition and restricted entry in arbitrage.

Fourth, there are also studies that are concerned with developing strategies for convergence bidders to maximize their profit. Given the purely financial nature of CB, the common assumption in this line of work is that CBs are small and do not have any major impact on the market prices. That is, the (implicit) assumption is that the convergence bidder is *price taker*. Thus, the focus has been on issues such as price prediction and stochastic learning to optimally place CBs under different risk management considerations, c.f. [23], [24].

Even though the study in this paper involves the formulation of the strategic convergence bidding, the scope of this paper is somewhat in between the studies in the third and the fourth group of papers that we listed above. On one hand, a key assumption in our analysis is that CBs are large enough to have impact on market prices. Therefore, our formulations involve not only the convergence bidding optimization but also the underlying market optimization problems. As a result, the analysis in this paper is fundamentally different from the strategic convergence bidding methods in the forth aforementioned group of papers. On the other hand, by solving the formulated convergence bidding problem we investigate the impact of strategic CBs on the market; with a viewpoint different from those in the third aforementioned group of papers. For example, unlike in analytical models in [18]–[20], [22], our focus here is on nodal electricity markets; therefore our analysis of strategic convergence bidding includes placing CBs in *multiple locations*. Other aspects that have not been previously investigated, but covered in this paper, include joint bidding with physical demand bids and physical supply bids as well as physical contingency in the power grid.

Last but not least, the related literature also includes the rich body of work on strategic bidding for various physical assets, such as for generation companies [25], [26], demand entities [27]–[29], as well as batteries and microgrids [30]–[32].

C. Summary of Contributions

The main contributions of this work can be listed as follows:

1) To the best of our knowledge, this is the first paper that provides the formulation and a methodology to solve the problem of strategic convergence bidding in nodal electricity markets. After identifying the main practical components in this problem, the bidding problem is mathematically formulated as a bi-level profit maximization problem. The initial problem is then transformed into a tractable mixed-integer linear program (MILP).
2) Multiple special cases are discussed and reflected in the problem formulation, such as *joint bidding* with physical assets, coordinated bidding at different locations, net-zero CBs, and convergence bidding with the social objective of closing the price gaps. The implication of each special case is studied. The long-term performance of strategic convergence bidding is also investigated.
3) We then asses the impact of optimal convergence bidding on the performance of nodal electricity market. Some of the insightful observations include: First, considering convergence bidding will not necessarily close the gap between the DAM and RTM prices. Second, physical assets may strategically loose money on their CBs to increase profit on their physical asset. Third, the possible adverse impact of strategies leveraged by convergence bidders to increase their profit on the price convergence of the RTM and DAM is illustrated.

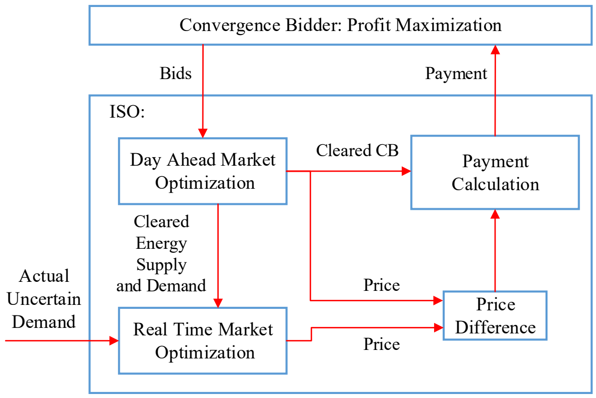

Fig. 1. The main components in the strategic convergence bidding problem.

II. Strategic Convergence Bidding Problem Formulation

Consider a convergence bidder that submits CBs in a wholesale electricity market aiming to maximize its own profit. The process and its main components are shown in Fig. 1. In this paper, our focus is on two-settlement electricity markets, such as those operated by CAISO and PJM. In these markets, DAM is cleared a day before the operating day. The DAM prices are determined and the generators and demands are scheduled. The awarded supply and demand CBs are also determined. Once the actual demand is revealed in the time of operation, the ISO balances the actual demand with the supply from the cleared generators in the DAM plus the supply bids in the RTM. Thus, it implements another market optimization taking the RTM supply bids into account, which determines the RTM prices. ISO then calculates the payment to convergence bidders based on the awarded CBs as well as the price difference between DAM and RTM. If the DAM price is higher (lower) than the RTM price, then supply (demand) CBs are profitable. Next, we explain the detailed models for the components in Fig. 1.

A. Market Model

1) Day-Ahead Market: Market participants can commit to buy or sell wholesale electricity one day before the operating day. Once ISO collects all the supply, demand and convergence bids, it solves the following optimization problem:

$$\textbf{min} \sum_t \Bigg[\sum_{j\in\mathcal{N}^{g,d}} a_j^{g,d}[t] P_j^{g,d}[t] - \sum_{j\in\mathcal{N}^{l,d}} a_j^{l,d}[t] P_j^{l,d}[t] + \sum_{i\in\mathcal{N}^b} a_i^{b,d}[t] P_i^{b,d}[t] \Bigg], \tag{1a}$$

s.t.

$$P_j^{g,d}[t] - P_j^{l,d}[t] + P_i^{b,d}[t] = \sum_{m|(n,m)\in\mathcal{L}} H_{nm}(\theta_n^d[t] - \theta_m^d[t]), \qquad : \lambda_n^d[t], \ \forall t, n \tag{1b}$$

$$0 \le P_j^{g,d}[t] \le \overline{P_j^g}[t], \qquad : \underline{\mu_j^{g,d}}[t], \overline{\mu_j^{g,d}}[t], \ \forall t, j \tag{1c}$$

$$0 \le P_j^{l,d}[t] \le \overline{P_j^l}[t], \qquad : \underline{\mu_j^{l,d}}[t], \overline{\mu_j^{l,d}}[t], \ \forall t, j \tag{1d}$$

$$-\overline{P_i^b}[t] \le P_i^{b,d}[t] \le \overline{P_i^b}[t], \qquad : \underline{\mu_i^b}[t], \overline{\mu_i^b}[t], \ \forall t, i \tag{1e}$$

$$-\overline{C_{nm}} \le H_{nm}(\theta_n^d[t] - \theta_m^d[t]) \le \overline{C_{nm}}, \quad : \underline{\beta_{nm}^d}[t], \overline{\beta_{nm}^d}[t], \ \forall t, (n,m), \tag{1f}$$

where, $P^{g,d}, P^{l,d}, P^{b,d}$ and θ^d are the primal decision variables. The dual variable for each constraint is shown on the right side of the colon symbol. Note that, $a^{b,d}$ which is the decision variable of the convergence bidder, acts as a parameter in the DAM optimization problem. The objective function (1a) is *the cost of energy offered by the producers*, minus *the revenue from supplying demand*, plus *the cost resulted from the convergence supply (demand) bids* in each market interval. Equation (1b) enforces the day ahead power balance at each bus. Constraints (1c)-(1e) represent the limits of the power offered by the generation units, the bounds of the demand and the CBs, respectively. Note that, $-\overline{P}^b$ refers to the maximum amount of the demand CB. Constraint (1f) imposes the capacity limits of each transmission line.

2) Real-Time Market: In the RTM, the differences between day ahead commitments and the actual real time demand is balanced. The RTM is performed just minutes before the actual power delivery of each producer, where the increased or decreased energy volume of each generator is determined to maintain the real time demand and supply balance of the system. The deviations of the actual generation and load from what were scheduled in the DAM are then settled at the RTM price. The RTM optimization problem considering the different scenarios at each time slot can be written as:

$$\textbf{min} \sum_k \Bigg[\sum_{j\in\mathcal{N}^{g,r}} a_{jk}^{g,r+}[t] P_{jk}^{g,r+}[t] - a_{jk}^{g,r-}[t] P_{jk}^{g,r-}[t] \Bigg] \tag{2a}$$

s.t.

$$P_j^{g,d}[t] - P_j^{l,d}[t] - \Delta_{jk}[t] + P_{jk}^{g,r+}[t] - P_{jk}^{g,r-}[t] = \sum_{m|(n,m)\in\mathcal{L}} H_{nm}(\theta_{nk}^r[t] - \theta_{mk}^r[t]), \quad : \lambda_{nk}^r[t], \ \forall k, n \tag{2b}$$

$$0 \le P_{jk}^{g,r+}[t] \le \overline{P_j^g}[t] - P_j^{g,d}[t] \quad : \underline{\mu_{jk}^{g,r+}}[t], \overline{\mu_{jk}^{g,r+}}[t], \ \forall k, j \tag{2c}$$

$$0 \le P_{jk}^{g,r-}[t] \le P_j^{g,d}[t] \quad : \underline{\mu_{jk}}^{g,r-}[t], \overline{\mu_{jk}^{g,r-}}[t], \ \forall k, j \tag{2d}$$

$$-\overline{C_{nm}} \le H_{nm}(\theta_{nk}^r[t] - \theta_{mk}^r[t]) \le \overline{C_{nm}}, \quad : \underline{\beta_{nmk}^r}[t], \overline{\beta_{nmk}^r}[t], \ \forall k, (n,m), \tag{2e}$$

where, $P^{g,r+}, P^{g,r-}$ and θ^r are the primal decision variables. The dual variables are also shown. The objective function (2a) minimizes the total cost of redispatching energy. Equation (2b) enforces the real time power balance at each bus. The actual demand is deviated by the random variable Δ from the cleared demand $P^{l,d}$ in the DAM. Constraints (2c)-(2d) represent the limits on the allowable deviation of the power generations in the real time from the day ahead schedule. Constraint (2e) imposes the capacity limits of each transmission line.

Since the DAM is cleared prior to the RTM, the decision variables $P^{g,d}, P^{l,d}$ in problem (1) act as parameters in problem (2). Also note that, only the cleared physical supply $P^{g,d}$ and the cleared physical demand $P^{l,d}$ appear in the power balance constraint (2b). This constraint does not include the cleared CBs. This is because, in practice, CBs' position is zeroed-out in the RTM process; because the RTM optimization is solely based on the *actual generation* and the *actual consumption*. It is worth adding that, although CBs do *not* explicitly appear in the RTM optimization problem, they *do* affect the schedule of physical resources at the DAM; therefore they *do* indirectly affect the RTM prices.

Since CBs must be decided one-day ahead of RTM, the main source of uncertainty here is in the RTM. RTM includes uncertainty since it is very close to the time of operation. Therefore, the randomness is taken to account only in RTM.

B. Optimal Convergence Bidding Problem

Suppose the convergence bidder of interest can submit both supply and demand CBs at some pre-determined nodes $i = 1, \dots, I$. Randomness is modeled by scenarios $k = 1, \dots, K$. For the convergence bidder to maximize its own profit, it must solve the following optimization problem:

$$\begin{aligned} \textbf{max} \quad & \sum_t \sum_{i \in \mathcal{N}^b} \left[\lambda_i^d[t] - \sum_k \rho_k \lambda_{ik}^r[t] \right] \times P_i^{b,d}[t] \\ \textbf{s.t} \quad & \text{Market Optimization Problems } (1), (2). \end{aligned} \tag{3}$$

where, $P_i^{b,d}$ is positive for a supply CB and negative for a demand CB. The objective function in (3) is the payment (or charge) to the convergence bidder, which is obtained by multiplying the cleared CBs, i.e., $P^{b,d}$, determined in the DAM, to the difference between the nodal DAM clearing price λ^d and the expected nodal RTM clearing price λ^r.

The optimization problem in (3) incorporates all the components in Fig. 1. This problem is a *bi-level* program, where the DAM optimization problem in (1) and the real time market optimization problem in (2) form the lower-level problems.

C. Special Cases

1) Joint Physical and Convergence Bids: Physical assets, such as generators, can leverage CBs to further increase their profit. While the constraints of the profit maximization problem remain as in (3), the objective function for joint physical and convergence bidding must change as follows:

$$\begin{aligned} \textbf{min} \sum_t \Bigg[& \sum_{i \in \mathcal{N}^b} \left(\lambda_i^d[t] - \sum_k \rho_k \lambda_{ik}^r[t] \right) \times P_i^{b,d}[t] \\ & + \sum_{i \in \mathcal{N}^{g,d}} \lambda_i^d[t] \times P_i^{g,d}[t] - f\left(P_i^{g,d}[t]\right) \Bigg], \end{aligned} \tag{4}$$

where $f(\cdot)$ denotes the operation cost of the generation unit. It should be noted that, if submitting CBs is not profitable for a physical asset, then the optimization problem in (4) results in a solution at which the CB quantity is zero. In other words, there is never a harm in using the formulation in (4); because the CB will be automatically and optimally set to zero, if needed.

2) Net-zero Convergence Bidding: This is one of the options available to a strategic convergence bidder to hedge against the uncertainty in the *energy components* of LMPs. The objective of a net-zero CB is to make profit from the *congestion component* of LMPs, as opposed to making profit from the fluctuations in the *energy component* of LMPs. The problem formulation for this strategy is similar to one in problem (3), with the following additional constraint:

$$\sum_{i \in \mathcal{N}^b} P_i^{b,d}[t] = 0, \tag{5}$$

where the net energy terms of demand and supply CBs is zero. Net-zero convergence bidding is also sometimes referred to as *up-to-congestion* bidding, such in the PJM market [9].

3) Price Gap Minimization: As far as an ISO is concerned, convergence bidding is ultimately a mechanism to close the price gap between the DAM and RTM. One may ask: *how is this concern aligned with or against the goal of a convergence bidder, i.e., maximizing its own profits?* Thus, we introduce a metric, called Summation of Price Differences (SPD), to measure how much the ISO's ultimate goal of closing the price gaps is achieved. This automatically results in introducing a special case, where the objective of the convergence bidder is to minimize SPD, instead of maximizing its own profile:

$$\textbf{min} \sum_t \sum_{i \in \mathcal{N}^b} \left| \lambda_i^d[t] - \sum_k \rho_k \lambda_{ik}^r[t] \right|. \tag{6}$$

4) Risk Management: While net-zero bidding can reduce the risk against uncertainty in the marginal cost of congestion; there are also ways to more explicitly address risk management in convergence bidding. For example, one option is to include Conditional Value at Risk (CVaR) either in the objective function or in the constraints [33], [34]. Other techniques, such as robust optimization, may also be used, c.f. [35]–[37]. In all methods, the ultimate goal is to provide the market participant with a mechanism to control the risk-profit trade-off.

III. Solution Method

The nonlinear bi-level optimization problem in (3) is difficult to solve. In fact, it is even more challenging compared to the standard bi-level optimization problems that often appear in market bidding problems, e.g., in [31], due to the presence of *two* lower-level problems that are *interconnected* in their variables. Nevertheless, we will next present a method to find the global optimal solution of problem (3) by converting it into a single mixed integer linear program (MILP), whose optimal solution can be procured with a reasonable computation time.

We start by first replacing the lower level problems (1) and (2) with their equivalent set of Karush-Kuhn-Tucker (KKT) conditions. Problem (1) has the following KKT conditions:

$$\text{Eqs. } (1b) - (1f) \tag{7a}$$

$$a_j^{g,d}[t] - \lambda_j^d[t] + \overline{\mu_j^{g,d}}[t] - \underline{\mu_j^{g,d}}[t] = 0 \qquad \forall t, j \tag{7b}$$

$$-a_j^{l,d}[t] + \lambda_j^d[t] + \overline{\mu_j^{l,d}}[t] - \underline{\mu_j^{l,d}}[t] = 0 \qquad \forall t, j \tag{7c}$$

$$a_i^{b,d}[t] - \lambda_i^d[t] + \overline{\mu_i^{b,d}}[t] - \underline{\mu_i^{b,d}}[t] = 0 \qquad \forall t, i \tag{7d}$$

TABLE I
TRANSMISSION LINE DATA

From	To	Reactance [p.u.]	From	To	Reactance [p.u.]
1	2	0.06	7	9	0.11
1	5	0.22	6	11	0.2
2	3	0.2	6	12	0.26
2	4	0.18	6	13	0.13
2	5	0.17	7	8	0.18
3	4	0.17	9	10	0.08
4	5	0.04	9	14	0.27
4	7	0.21	10	11	0.19
4	9	0.56	12	13	0.2
5	6	0.25	13	14	0.35

TABLE II
SUBMITTED SUPPLY BIDS TO THE DAM [$/MWH]

Bus	a^1	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9	a^{10}
1	14	18	22	26	30	34	38	42	46	50
2	12	17	22	27	32	37	42	47	52	57
3	17	20	23	26	29	32	35	38	41	44

TABLE III
SUBMITTED SUPPLY BIDS TO THE RTM [$/MWH]

Bus	a^1	a^2	a^3	a^4	a^5	a^6
6	15	23	31	39	47	55
8	5	14	23	32	41	50

$$\sum_{m \mid (n,m)\in\mathcal{L}} \Big[H_{nm}\left(\lambda^d_n[t] - \lambda^d_m[t]\right) + H_{nm}\left(\overline{\beta^d_{nm}}[t] - \overline{\beta^d_{mn}}[t]\right) + H_{nm}(\underline{\beta^d_{mn}}[t] - \underline{\beta^d_{nm}}[t])\Big] = 0 \quad \forall t, n \tag{7e}$$

$$X^d . * Y^d = 0 \tag{7f}$$

$$Y^d \geq 0, \tag{7g}$$

where (7b)-(7e) are the gradient equilibrium conditions and (7f) enforces complementarity slackness.

Similarly, the KKT conditions for problem (2) become:

$$\text{Eqs. } (2b) - (2e) \tag{8a}$$

$$a^{g,r+}_{jk}[t] - \lambda^r_{jk}[t] + \overline{\mu^{g,r+}_{jk}}[t] - \underline{\mu^{g,r+}_{jk}}[t] = 0 \quad \forall k, t, j \tag{8b}$$

$$-a^{g,r-}_{jk}[t] + \lambda^r_{jk}[t] + \overline{\mu^{g,r-}_{jk}}[t] - \underline{\mu^{g,r-}_{jk}}[t] = 0 \;\; \forall k, t, j \tag{8c}$$

$$\sum_{m \mid (m,n)\in\mathcal{L}} \Big[H_{nm}\left(\lambda^r_{nk}[t] - \lambda^r_{mk}[t]\right) + H_{nm}\left(\overline{\beta^r_{nmk}}[t] - \overline{\beta^r_{mnk}}[t]\right) + H_{nm}\left(\underline{\beta^r_{mnk}}[t] - \underline{\beta_{nmk}}^r[t]\right)\Big] = 0 \quad \forall k, t, n \tag{8d}$$

$$X^r . * Y^r = 0 \tag{8e}$$

$$Y^r \geq 0. \tag{8f}$$

By replacing the lower level linear programs in (1) and (2) with the constraints in (7) and (8), problem (3) becomes a standard mathematical program with equilibrium constraint (MPEC):

$$\begin{aligned} \textbf{max} \; & \sum_t \sum_{i\in\mathcal{N}^b} \left[\lambda^d_i[t] - \sum_k \rho_k \lambda^r_{ik}[t]\right] \times P^{b,d}_i[t] \\ \textbf{s.t} \; & \text{Eqs. } (7), (8). \end{aligned} \tag{9}$$

The optimization problem in (9) is nonlinear; but it can be linearized and converted into a MILP. The details on the problem reformulation are explained in the Appendix.

IV. CASE STUDIES

We investigate the merit of the proposed analysis based on the IEEE 14-bus test system [38]. This network includes 20 transmission lines, where $\mathcal{N}^{g,d} = \{1, 2, 3\}$, $\mathcal{N}^{g,r} = \{6, 8\}$, $\mathcal{N}^{l,d} = \{2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14\}$, $\mathcal{N}^b = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$. The grid information and the offered data are shown in Tables I-IV. The transmission capacity of all transmission lines is 300 MW, except for line (1,5) and line (4,5) whose transmission capacity is 31 MW and 23 MW, respectively. Here the maximum limitation for CBs is 10 MWh unless otherwise stated. The corresponding MILP problem is solved using Gurobi 8 (http://www.gurobi.com).

A. Illustrative Example

Table V shows the DAM and RTM prices as well as the change in price gap *with* and *without* optimal CBs. Applying CB can help one LMP to converge in a bus, at the same time, causes the LMP divergence in another bus. Thus, we use SPD, see Section II.C.3, as the criteria to compare impact on price difference. By maximizing the profit of convergence bidder, SPD is increased to $226.79 from $203.89, which is *not* expected from CBs' outcome in electricity markets.

B. Uncertainty in Demand and Generation Bidding in RTM

One reason to submit CB is to hedge against the uncertainty in electricity markets. In this section, the uncertainties in the demand in RTM as well as the uncertainties in the generation bids that participate in RTM are considered. Monte Carlo simulation is used to generate 3000 scenarios. To keep computation tractable, we reduced the number of scenarios to 10 distinguished scenarios using the fast backward/forward method [39], [40]. The probabilities of the scenarios are 0.60, 0.12, 0.07, 0.05, 0.04, 0.03, 0.02, 0.03, 0.03, 0.01. In the stochastic case, the expected profit for convergence bidder is decreased to $113.25 from $150.42 in the deterministic case. The expected SPD is $182.06 in the presence of CBs, while it is $176.18 in the absence of CBs. The increase in SPD indicates that the profit maximization approach of

TABLE IV
DEMAND DATA

Bus	Demand in DAM [MWh]	Demand Change in RTM [MWh]
2	17.36	4.34
3	75.36	18.84
4	38.24	9.56
5	6.08	1.52
6	8.96	2.24
9	23.6	5.9
10	7.2	1.8
11	2.8	0.7
12	4.88	1.22
13	10.8	2.7
14	11.92	2.98

TABLE V
LMPs IN TWO-SETTLEMENT MARKETS WITHOUT CBs, AS WELL AS THE OPTIMAL CBs AND CHANGE IN PRICE GAPS UNDER STRATEGIC BIDDING

Bus No.	Without CB		With CB	
	λ_i^d	λ_i^r	$P_i^{b,d}$	Change in Price gap
1	38	23.52	9.62	-4
2	39.92	25.39	-4.88	-1.44
3	41	30.8	-9.88	0
4	41.92	35.39	9.12	1.22
5	42.55	16.65	-0.19	2.06
6	42.33	23	0.37	1.78
7	42.03	32	-1.38	1.38
8	42.03	32	-0.13	1.38
9	42.09	30.22	-0.12	1.46
10	42.13	28.99	0.63	1.51
11	42.23	26.07	-0.38	1.64
12	42.31	23.57	5.37	1.76
13	42.3	24.01	0.12	1.73
14	42.18	27.52	-0.13	1.58

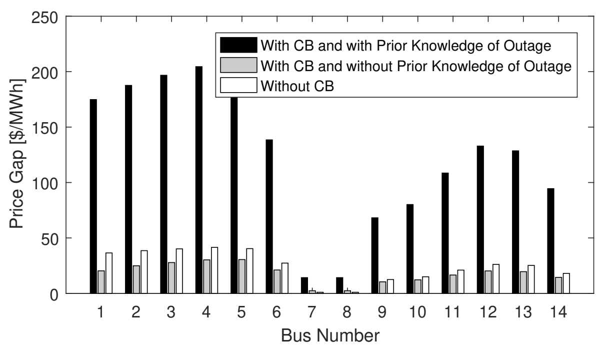

Fig. 2. The price gap in various buses within the network under a contingency.

convergence bidders increases, i.e., worsens, the price gap between the DAM and RTM. Thus, convergence bidders make profit despite the fact that they exacerbate the price gap.

C. Contingency within Power Network

Power grid is subjected to various contingencies; and it is interesting to find out whether CBs can heal or exacerbate the situation when contingencies occur. Consider a contingency in the RTM in the form of an outage in the transmission line between buses 4 and 7. The results are shown in Fig. 2. Two scenarios are considered here. In the first scenario, it is assumed that convergence bidders are *unaware* of the contingency when they submit their bids to the DAM. In the second scenario, it is assumed that they are *aware* of the situation. Under the first scenario, the profit for convergence bidder *increases*, from \$150.42 under normal conditions to \$350.75 under the contingency. Here, CBs lead to an *improvement* in the price convergence as seen by reduction in SPD to \$253.27 from \$344,42 in the normal case.

Under the second scenario, the profit for convergence bidders increase drastically, from \$150.42 under normal conditions to \$3,133.78 under the contingency. However, we can now see undesirable *divergence* of LMPs between the two markets. Specifically, SPD increases to \$1743.59. Thus, the increase in the profit of convergence bidder comes with a retrogression in price convergence between the two markets.

D. Joint Physical and Convergence Bids

In this section, we investigate the scenario where CBs are submitted *in coordination with physical bids*. We will consider two different cases. First, the case where CBs are submitted jointly with physical supply bids. Second, the case where CBs are submitted jointly with physical demand bids.

1) Physical Supply Bids: CBs are assumed to be submitted one at a time by each individual generator. Generators G1, G2, and G3, submit physical bids to the DAM; and generators G6 and G8 submit physical bids to the RTM. The profit of each generator while submitting CBs to maximize its profit is shown along with the one without CB in Fig. 3. We observe that submitting CBs can serve as a *leverage* for generators to significantly *increase* their profit. For example, by submitting not only physical bids but also CBs, generator G3 increases its profit from \$1080 without CB to \$5776.76 with CBs even though it loses \$1481.79 on CB. The profit of generator G8 that bids in the RTM also increases from \$540 without CB to \$3736.09 with CB even though it loses \$1784.78 on CBs. These two units strategically lose money on their CBs to significantly increase their profit on physical supply bids.

We investigate the awarded CBs as well as the profit from CBs for generators G3 and G8 in order to find the cause of the significant increase in the profit. As shown in Fig. 4, the profit of generators G3 and G8 are negative from the submitted CB at all buses with the exception of buses 1, 2, 3, and 13 for generator G3, and buses 5 and 6 for generator G8. It is interesting to note that the CBs submitted by generator G3 cause an increase in the price gap between the two markets at the *same* buses that their profit from submitting CBs is negative when it is compared with a scenario with only physical bids. There is one exception which is the generator bus i.e. bus 3. It is the ultimate desire of generator G3 to increase the LMP on bus 3 to increase its profit from its physical bids in the DAM. On the other hand, the CBs submitted by generator G8 cause a change in the sign of price gap. Thus, the LMPs within the RTM become larger than those in the DAM. Taking a closer look at buses 7-12 reveals that the CBs submitted by generators G3 and G8 are demand bid and supply bid, respectively; and the changes in price gap are in the opposite direction of profit making for these CBs. However, as a result of this strategy, the LMP on bus 3 in the DAM is increased from \$41 to \$102.68 which leads to the notable increase in the profit of generator G3 that would compensate for the loss from its CBs. Similarly, the LMP on bus in the RTM is increased from \$32 to \$117.43 which also leads to the notable increase in the profit of generator G8 to the level that will cover the loss from submitted CBs.

It is interesting also to check the impact of CBs on the awarded generation dispatch of each generator, as shown in Fig. 3. For the units that participate in the RTM, the awarded dispatch is almost doubled with the submission of CBs. However, there are not significant changes in the awarded generation dispatch of units that participate in the DAM. Among those units, the largest increase belongs to G3 which also has the largest increase in the profit from CBs.

The above results can be interpreted also in the context of

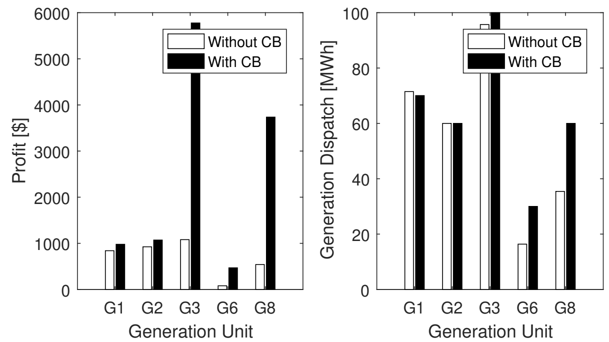

Fig. 3. Total profit and awarded dispatch of generators *with* and *without* CBs.

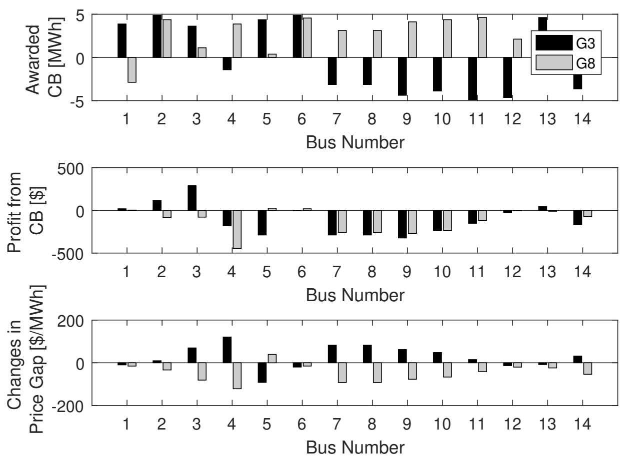

Fig. 4. The profit from CB on each bus while submitted by various generators.

market power. First, consider the results for Generator G1. By submitting CBs, Generator G1 *reduced its physical generation yet it increased its total profit*. Next, consider Generators G3 and G8. By submitting CBs, they both *significantly increased their total profit*; but they both *lost money on their CB market participation*. That means, their CB market participation was not meant to gain profit in the CB market; but rather to create a scenario to put them in a much better position to gain a huge profit on their physical bids. The examples suggest that convergence bidding by a physical asset owner may increase the market power of that physical asset owner.

2) Physical Demand Bids: Load service entities (LSEs) who bid in the market to purchase electricity may also submit CB in order to reduce their payments. In this section, we investigate the impact of CBs submitted by LSEs on their payment. The total payment of LSEs *reduced* from $10134.72 when they do not submit CB to $9373.06 once they strategically submit CBs. The quantities of submitted CBs on each bus that led to the *decrease* in its LMP in both DAM and RTM as well as the breakdown of the $844.52 decrease in the payment of LSEs are given in Table VI. Submitting CBs by LSEs led to the *decrease* in LMPs of all buses in the DAM. Similarly, CBs caused the *decrease* in LMPs of the majority of buses in the RTM, with the exception of Bus 3. However, the second largest decrease in the payment of LSEs is on Bus 3 with $121.02 decrease in the payment. It is interesting to observe that as a convergence

TABLE VI
IMPACT OF CBS SUBMITTED BY LSES

Bus ID	Changes in LMPs of the DAM [$/MWh]	Change in LMPs of the RTM [$/MWh]	Changes in Payments of LSEs [$]	Quantity of Submitted CB [MWh]
1	0	-7.54	0	-4.88
2	-1.92	-5.87	-58.807	-3.38
3	-3	-1.07	-246.2388	-4.13
4	-3.92	3.02	-121.0296	0.87
5	-4.55	-13.64	-48.3968	-0.93
6	-4.33	-8	-56.7168	4.62
7	-4.03	0	0	0.37
8	-4.03	0	0	4.37
9	-4.09	-1.58	-105.846	4.62
10	-4.13	-2.67	-34.542	-1.88
11	-4.23	-5.26	-15.526	-3.88
12	-4.31	-7.49	-30.1706	1.12
13	-4.3	-7.09	-65.583	3.37
14	-4.18	-3.98	-61.686	-2.13

bidder, LSEs are not necessarily submitting demand CBs and they are also submitting supply CBs e.g. 4.62 MWh submitted as a CB on bus 9. Regardless of the type of CB, the aim of LSEs is to decrease the LMPs in both DAM and RTM to lower their payment. Interestingly, the profit of LSEs from their CB is negative. This reveals that LSEs tolerated a loss from their CBs to reduce their payment on their physical demand.

Submitting CBs by LSEs results in more decrease in LMPs in the RTM than in the DAM, as one can see in Table VI.

E. Net-Zero Bidding Plan

By taking a net-zero bidding plan, speculators hedge against the uncertainty in changes in the energy component of LMPs. The profit of the convergence bidder without and with adopting the net-zero bidding plan is $150.42 and $70.13 An interesting observation is shown in Table VII with respect to the price gap. We can see that once the net-zero bidding plan is adopted, the price gap either reduces or remains unchanged. The only exception is at buses 1 and 2 where the price gap becomes *larger* due to net-zero convergence bidding. These two buses are connected to generators G1 and G2. The generation dispatch of G1 increases from 58.36 MWh to 60.4 MWh and the generation dispatch of G3 increases from 80.64 MWh to 87.06 MWh. The generation dispatch of G2 remains unchanged.

Here the observation is that the awarded generation dispatch of units that bid in the RTM decreases when net-zero bids are submitted. Here, the generation dispatch of G6 decreases from 20 MWh to 17.6 MWh and the generation dispatch of G8 decreases from 40 MWh to 34.34 MWh for G8. It is also interesting to note that by adopting the net-zero bidding plan, the power flow in transmission lines are changed in the DAM and the RTM. For example, the congestion occurred in the line between buses 4 and 5 in the DAM does not actually occurs in the RTM. This is an example of how convergence bidders take advantage of *congestion component* of LMPs to make profit.

It is insightful to compare the effectiveness of CBs *with* and *without* adopting the net-zero bidding plan. Without net-zero bidding, SPD is $215.95; while, it decreases to $203.89 when the net-zero bidding plan is adopted. Thus, adopting net-zero

TABLE VII
THE OPTIMAL CBS AND PRICE GAPS WITH AND WITHOUT NET-ZERO PLAN

Bus No.	With CB (No Net-zero)		With CB (Net-zero)	
	$P_i^{b,d}$	price gap	$P_i^{b,d}$	price gap
1	9.62	10.48	0.31	10.48
2	-4.88	13.09	5.12	13.09
3	-9.88	10.2	-5.63	10.2
4	9.12	7.75	2.87	7.75
5	-0.19	27.96	9.94	27.96
6	0.37	21.11	6.12	21.11
7	-1.38	11.41	8.87	11.41
8	-0.13	11.41	8.87	11.41
9	-0.12	13.33	-9.63	13.33
10	0.63	14.65	-5.88	14.65
11	-0.38	17.8	-6.88	17.8
12	5.37	20.5	-7.88	20.5
13	0.12	20.02	-6.63	20.02
14	-0.13	16.24	0.37	16.24

bidding plan by convergence bidders leads to a better price convergence compared to the one without this plan.

Another interesting observation is when the net-zero bidding plan is adopted in the stochastic case, where the expected profit reduces from \$113.25 to \$10.78; and SPD indicates an increase in price gap. For example, the expected SPD increases to \$58.64. Thus, net-zero bidding will not only decrease the profit for convergence bidder it will also worsen the price gap.

F. Profit Maximization vs Price Gap Minimization

Recall from Section I that, as far as an ISO is concerned, convergence bidding is a mechanism to close the gap in prices between the DAM and RTM, i.e., to solve the problem in (6). In this section, we seek to investigate how such goal is aligned with the objective of a convergence bidder to maximize its own profit, i.e., to solve the optimization problem in (3).

Under the profit maximization framework, as we increase the maximum allowed convergence bidding quantity at each bus from 1 MWh to 25 MWh, the convergence bidder's profit *increases*, see the curve with the star markers in Fig. 5(a); while the price gap initially decreases but later increases; see the curve with the star markers in Fig. 5(b). Therefore, the increase in the profit of the convergence bidder may or may not coincide with improvement in the price gap between the two markets, depending on the size of the CB. It should be noted that the monotone behavior in the curve in Fig. 5(a) is due to the fact that the optimal objective value in the profit maximization problem in (3) is monotonically non-decreasing with respect to the constraint on maximum CB quantity. This profit curve ultimately reaches its maximum limit at \$104.3.

Under the price gap minimization framework, as we increase the maximum allowed convergence bidding quantity at each bus from 1 MWh to 25 MWh, price gap across the buses in the network *decreases*, see the curve with the square markers in Fig. 5(b); while the convergence bidder's profit fluctuates; see the curve with the square markers in Fig. 5(a). Therefore, in this case, the decrease in the price gap may or may not coincide with the improvement in the convergence bidder's profit, depending on the size of the CB. It should be noted that the monotone behavior in the curve in Fig. 5(b) is due to the fact that the optimal objective value in the price gap minimization problem in (6) is monotonically non-increasing with respect to the constraint on maximum CB quantity. This SPD curve ultimately reaches its minimum limit at zero.

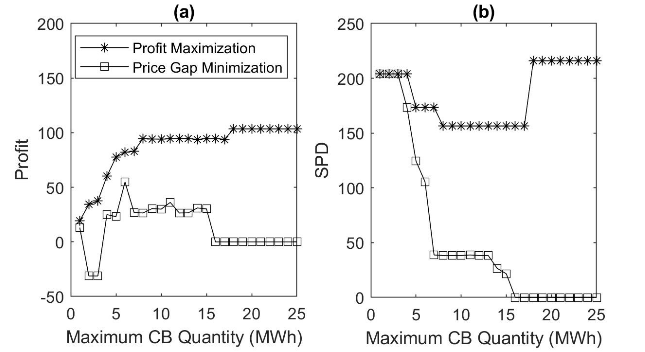

Fig. 5. Comparing profit maximization and price gap minimization strategies versus maximum allowable CB: (a) Convergence bidder's profit; (b) Price gap.

G. Long-Term Bidding Performance

In this section, we demonstrate the long-term performance of strategic convergence bidding over a period of one month. The results are shown in Fig. 6(a) and (b). The base demand is set according to the normalized hourly load of California ISO between March 10, 2020 and April 9, 2020. Both normal operating conditions as well as operation under physical contingencies are considered. In particular, four days are assumed to include physical contingencies, as marked on the figure. The physical contingencies are in form of transmission line tripping on transmission lines 8, 12, 18, and 20, respectively.

The total daily profit of the convergence bidder under the profit maximization paradigm and the daily average SPD across the network under the price gap minimization paradigm are shown in Figs. 6(a) and (b), respectively. We can observe that the varying load conditions can result in varying profit levels and varying SPD levels. However, the basic concepts in strategic convergence bidding remain the same, including under both normal conditions and during physical contingencies. For example, market uncertainty may cause financial loss at certain hours; but as we can see Figs. 6(a), the total daily profit always remains positive and considerable. As another example, similar to the results in Section IV-C, if the convergence bidder is not aware of the physical contingency then it can perform as well compared to the case that it is aware of the physical contingency such as due to a planned maintenance.

V. CONCLUSIONS

The strategic behavior of speculators who submit CBs in a two-stage electricity market is investigated. Five research questions are raised and answered. *First*, the optimal convergence bidding problem is formulated from the viewpoint of a strategic market player whose goal is to maximize its own profit. The formulated problem is bi-level, where the problem at the lower-level captures the operation of both day-ahead and real-time markets. The non-convex bi-level problem is reformulated to a mixed-integer linear program suitable for the off-the-shelf solvers. *Second*, it is shown that if the

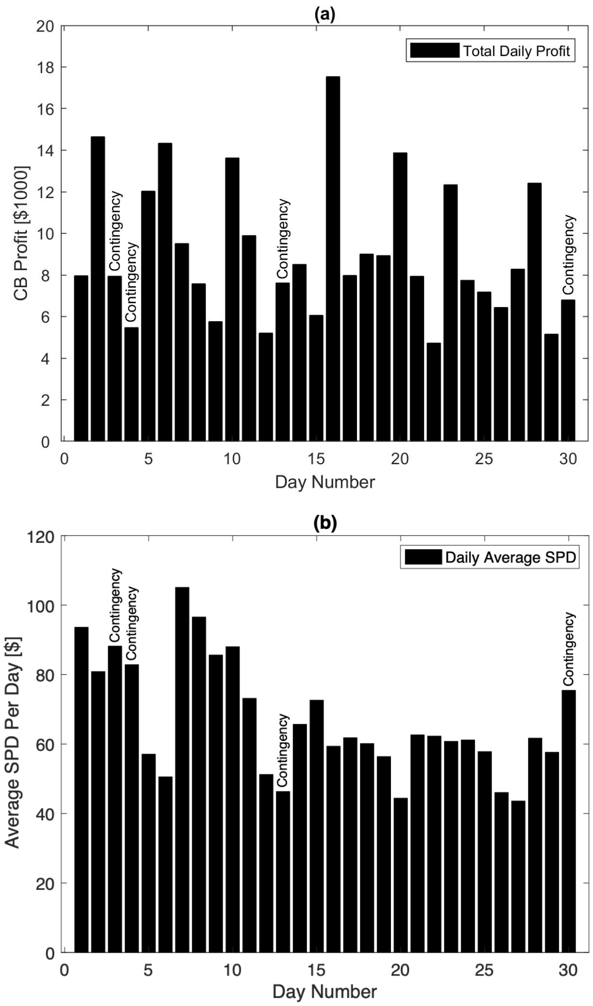

Fig. 6. Long-term convergence bidding performance over a period of one month under stochastic market conditions and in presence of multiple physical contingencies: (a) Total daily profit of the convergence bidder under CB profit maximization; (b) Daily average SPD under price gap minimization.

convergence bids are submitted by a market participant that owns physical asset, then strategic bidding can be coordinated between physical bids and convergence bids. Under such coordination, convergence bids can be used as a leverage by generators to increase their overall profit. *Third*, it is observed that a strategic convergence bidder that aims to maximize its profit is not necessarily in line with the intended purpose of CBs to lower the gap across the day-ahead and real-time markets. In fact, while strategic behavior of convergence bidder can lower the price gap in many occasions, we can demonstrate scenarios where there is a trade-off between the interest of the system operator to lower the gap across the two markets and the interest of the convergence bidder to gain profit. *Fourth*, the impact of strategic bidding on price gap can be different at different group of buses. Furthermore, increasing the size of convergence bids may decrease or increase the price depending on the operating conditions. *Fifth*, uncertainty in demand can affect both the convergence bidder's profile as well as its impact on the market. In the event of contingency, depending on the prior knowledge of the convergence bidder on the event, the gap might shrink or stretch. Also, while adopting net-zero bidding plan lowers the gap in the prices across the two markets, it worsens once the uncertainty in demand is taken into consideration.

The study in this paper can be extended in various directions. First, the results here can be used to set forth policies by ISOs to control the impact of CBs on electricity markets. For example, policies can be designed to control the extent that owners of physical assets are allowed to participate in convergence bidding. Second, while the analytical framework that we designed in this paper is meant to be generic; it is for the most part based on how convergence bidding works in the California ISO market. The differences across different ISO markets and their possible impact on the operation of CBs can be investigated in the future. Third, other methods can be used to deal with the non-linearity and non-convexity in the formulated strategic convergence bidding problem, such as based on convex relaxation. Fourth, it is interesting to also investigate the possibility of forming games among convergence bidding market participants, including among those market participants that own physical resources, and the corresponding equilibrium and its characteristics.

Appendix: MILP Formulation

The optimization problem in (9) is non-linear due to its MPEC structure, where the nonlinearities come from three main sources; we address each issue individually next.

First, consider the *bilinear term* $\sum_i \lambda_i^d[t] P_i^{b,d}[t]$ in the objective function. Both $\lambda_i^d[t]$ and $P_i^{b,d}[t]$ are directly related to the DAM optimization in (1), which is a linear program and therefore has zero duality gap. Since strong duality holds for optimization problem (1), its optimal primal objective value is equal to its optimal dual objective value; and after reordering the terms, we can write the term $\sum_i \lambda_i^d[t] P_i^{b,d}[t]$ as:

$$\begin{aligned} &-\sum_{j\in\mathcal{N}^{g,d}} a_j^{g,d}[t] P_j^{g,d}[t] + \sum_{j\in\mathcal{N}^{l,d}} a_j^{l,d}[t] P_j^{l,d}[t] \\ &-\sum_{j\in\mathcal{N}^{l,d}} \overline{\mu_j^{l,d}}[t]\, \overline{P_j^{l}}[t] - \sum_{j\in\mathcal{N}^{g,d}} \overline{\mu_j^{g,d}}[t]\, \overline{P_j^{g}}[t] \\ &-\sum_{(n,m)\in\mathcal{L}} \overline{C_{nm}}\, \underline{\beta_{nm}^d}[t] - \sum_{(n,m)\in\mathcal{L}} \overline{C_{nm}}\, \overline{\beta_{nm}^d}[t]. \end{aligned} \quad (10)$$

Second, consider the *bilinear term* $\lambda_{ik}^r[t] P_i^{b,d}[t]$ in the objective function. Note that, variables $\lambda_{ik}^r[t]$ and $P_i^{b,d}[t]$ comes from *two separate* lower-level problems, and $P_i^{b,d}[t]$ does *not* exist in the RTM optimization. Therefore, strong duality does not help in this second case. We instead used the binary expansion method, where bi-linear terms involving two continuous variables are linearized using binary approximations of one variable combined with additional linear constraints, c.f., [41].

Finally, the complimentary slackness constraints $X^d.*Y^d$ and $X^r.*Y^r$ are also nonlinear; however, one can introduce new binary variables and apply the large M method to rewrite them in form of the following equivalent MILP constraints:

$$\begin{aligned} &0 \le X^d \le M z^d, 0 \le Y^d \le M(1-z^d), \\ &0 \le X^r \le M z^r, 0 \le Y^r \le M(1-z^r), \end{aligned} \quad (11)$$

where z^d and z^r are the vectors of auxiliary binary variables.

By taking the above three steps, one can convert the NLP in (9) into a MILP. The decision variables in such MILP include the prices of CBs as well as all the primal and dual variables of the DAM and RTM optimization models.

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